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Real Options in a Ramsey style Growth Model*

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Abstract. This paper studies the aggregate implications of microeconomic investment irreversibility and idiosyncratic uncertainty in a simple growth model by highlighting real option effects. We endogenize the drift rate of real option by connecting it to the state of the economy. Thereby, we extend the analysis of the optimal capital accumulation policy in the firm sector and show the different implications of idiosyncratic and aggregate uncertainty on growth dynamics.
Keywords. Irreversible investment; Idiosyncratic uncertainty; real options; growth
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1 Introduction

Investment in capital goods is often irreversible. A growing body of literature[1] has shown that on the background of irreversibility uncertainty induces a higher investment threshold for firms by generating a value of delaying investment decisions known as real option values.[2] Originally being a concept for explaining a firm's investment decision in micro finance theory, real options nowadays play an important role in the field of aggregate investment models. However, the discussion about the consequences of real options for the economy as a whole, that is, in general equilibrium is still in an infant state lacking a simple tractable model.[3] Analyzing the consequences of real options is not only important for a better understanding of the process of capital accumulation and economic growth, it is also significant for a better understanding of the dynamics of the business cycle in the light of uncertainty. Given the broad empirical discussion about the link between volatility, investment and growth, this represents a major theoretical shortfall.[4]

In this paper, we provide a tractable model that integrates real option effects into a simple general equilibrium model. In our model, firms make decisions about their capital stock in an uncertain environment, determined by volatile business conditions. Together with irreversibility of once invested capital this generates real option values in investment decisions.[5] On the one hand, our model connects a real options enhanced micro-foundation in the firm sector with utility maximizing households. On the other hand, it links the development of business conditions to the overall state of the economy, thereby endogenizing real options values. To further investigate the consequences of different kinds of volatility in our model, we introduce two sources of fluctuations - on the level of idiosyncratic demand and on the aggregate level.

Our analysis provides three main contributions. First, it offers an analytical tractable solution of real option effects in a general equilibrium setting. Second, our paper extends and generalizes partial equilibrium results by putting real options in a broader setting. Thereby, earlier findings on real option effects are confirmed on a general level. Most

[1] Most notable is the seminal work of Dixit and Pindyck (1994) in their book *Investment under Uncertainty*.

[2] McDonald and Siegel show that real options could almost double the demanded net present value to trigger investment. See (McDonald and Siegel 1986: p.708)

[3] Recently, Bloom et al. (2012) and Bachmann and Bayer (2013) have integrated real option values in DSGE models.

[4] See e.g. Ramey and Ramey (1995)

[5] This kind of firm's investment behavior is commonly used in partial investment models. See e.g. Abel and Eberly (1996); Abel and Eberly (1999); Alvarez (2011); Bloom (2000).

importantly, that real options do not influence the steady-state growth rate,[6] that the effect on the steady-state level of the capital stock is ambiguous [7] and that real options influence adjustment dynamics by inducing a hysteresis momentum.[8] Third, the endogenous reaction of real option values permits us to distinguish between the dynamic responses to different sources of shocks (firm or macro level) on aggregate investment even though both enter ex-post firm's investment decisions in the same way. In particular, our results show that aggregate shocks induce an endogenous dynamic reaction of the drift rate driving real option values. Treating the drift rate as constant and exogenous, as most of the real option literature does, leads imprecise results such as an overestimation of the hysteresis effect in the aftermath of temporary negative aggregate shocks.

This paper is mostly related to two strands of literature. On the one hand to the literature on irreversible investment under uncertainty in micro economic finance models (Dixit and Pindyck (1994); Abel and Eberly (1999)) and on the other hand to the literature on real option effects in aggregate macro models (Bertola (1994); Bloom (2009); Bloom et al. (2012); Bachmann and Bayer (2013)). In general, the implications of real options in investment decisions on the micro level are nowadays theoretically and empirically well understood.[9] However, although the importance of real option effects for aggregate investment has been confirmed in various empirical studies (Bloom (2009); Bloom et al. (2012); Caballero (1999); Bachmann et al. (2013)), a generalization of real option implications in a simple growth model has not been satisfactorily achieved yet. Earlier works of Bertola (1988), Bentolila and Bertola (1990) and Bertola and Caballero (1994) show that real options are important in shaping the dynamics of aggregate investment. More recently, Bloom (2009) has used real option effects with shocks to volatility showing that real options play a significant role in shaping real business cycles. Nevertheless, these partial equilibrium models lack important dynamic effects and do not offer general implication, such as effects on growth. Earlier attempts of integrating real option effects into a growth model can be found in Bertola (1994) and Jamet (2004). However, Bertola (1994) studies the balanced-growth equilibrium in a model with only two states of nature. Furthermore, he highlights labor adjustment costs which are quantitatively less relevant compared to constraints on capital adjustment. Jamet (2004) studies growth in a setting with uncertainty only on the macro level concentrating on

[6] Jamet (2004); Bloom (2000).

[7] Abel and Eberly (1996); Abel and Eberly (1999); Alvarez (2011).

[8] E.g. Dixit (1989);Dixit (1992); Dixit and Pindyck (1994).

[9] For an early assessment of the impact of the theory see Hubbard's review of Dixit and Pindyck's book *Investment under Uncertainty.* Hubbard (1994). Or Ingersoll and Ross (1992).

firm sector heterogeneity. By adopting the approach of Bloom (2009), both Bloom et al. (2012) and Bachmann and Bayer (2013) show the effect of uncertainty shocks in a business cycle model generating semi-endogenous real option values. However, as DSGE models they do not offer a tractable solution like our model. In addition, our results show that they are potentially ignoring an important part of real options dynamics by treating the drift rate of business conditions as exogenous.

The structure of the paper is as follows: section 2 introduces the micro economic foundation of the Ramsey style growth model augmented by irreversible capital investment. In section 3 we proceed to study the relevance of irreversibility on the aggregate investment level and show general implications for growth and adjustment dynamics, especially highlighting the different impact of idiosyncratic and macroeconomic uncertainty. Section 4 quantifies the impact of real options by showing the aggregate investment reaction to exogenous shocks in a simulation with a firm sector consisting of 2000 firms. In section 5 we discuss potential extensions and research applications of the model. Section 6 concludes.

2 Optimal Investment Decisions under Uncertainty

2.1 Households

An economy consists of a large number of identical risk-neutral infinite horizon households. Household's utility depends positively on consumption over time. It takes the simple form:

$$U = \int lnC_t e^{-\rho t} dt \tag{1}$$

where C_t is consumption in time period t and ρ represents household's time preference. The change in consumption can be expressed by the condition:

$$\frac{\dot{C}}{C} = r - \rho \tag{2}$$

where r defines the return on available investment possibilities to the households. Households offer their labor on the labor market. In addition, they own firms by holding a fraction of $\frac{1}{N}$ of every firm. Firm's profits are distributed to the households in terms of dividend payments. Because households own the firms, investment policies can be expressed in terms of rational decision making of a representative household based on its utility function.

2.2 Firms

The firm sector consists of a large number of M infinitely small firms. Individual output of the $i-th$ firm is produced by a combination of labor, capital and harrod-neutral technology. Technology is exogenous and identical to all firms. The production function takes the form:

$$Y_{it} = (A_t L_{it})^{1-\alpha} K_{it}^{\alpha} \qquad \text{with} \quad i = 1, ..., M \tag{3}$$

where Y_{it} denotes the production of the $i-th$ firm at time t. α denotes the constant capital share. Firms face an isoelastic demand curve where demand for its individual good depends on the price of the produced good P_{it}, aggregate output Y_{At} and idiosyncratic preference Z_{it}. Aggregate output and idiosyncratic preference are (possibly) uncertain and follow a geometric Brownian motion. The stochastic idiosyncratic preference are drawn from the same distribution for all firms.

$$\text{Macro level uncertainty:} \quad \frac{dY_{At}}{Y_{At}} = \mu_A dt + \sigma_A dW_1(t) \tag{4}$$

$$\text{Firm level uncertainty:} \quad \frac{dZ_{it}}{Z_{it}} = \sigma_Z dW_2(t) \tag{5}$$

The drift rate of the macro level in eq. (4) defines the (expected) growth rate of aggregate output. Given a Cobb-Douglas-production function with harrod-neutral technological progress as in eq. (3), this 'drift-rate' equals the growth rate of technology at its equilibrium. Both shocks are normally distributed with a mean of 1 and a variance of σ_Z, σ_A. We combine aggregated and idiosyncratic demand shifts into a single composite:[10]

$$X_{it} = Y_{At} Z_{it} \tag{6}$$

so overall demand for a representative firm takes the form:

$$Y_{it} = X_{it} P_{it}^{-\epsilon} \qquad \text{with} \quad \epsilon > 1 \tag{7}$$

[10] Here, we follow Bloom (2009) in creating a composite volatility function. Different to our paper Bloom also includes unit level uncertainty basically to generate a better fit to firm's investment series.

$$\text{and} \quad \frac{dX_{it}}{X_{it}} = \mu_A dt + \sigma_X dW_3(t) \quad \text{with} \quad \sigma_X > 0 \quad \text{and} \quad X_0 > 0 \tag{8}$$

In eq. (7) ϵ denotes the price elasticity of demand. Eq. (8) shows that the change in overall demand for the good of a single firm is assumed to follow a geometric Brownian motion as well. This will be true if either the volatility on the macro level or on the firm level will be equal to zero or when both volatilities are exactly the same.[11] In eq. (4), (5) and (8) $W_{1,2,3}(t)$ are the increment of a Wiener process with $E(dW_{1,2,3}(t)) = 0$ and $(W_{1,2,3}(t))^2 = dt$. The exogenous drift rate μ_A represents the growth rate of technology or, from the point of view of the firms, the drift rate of business conditions in the equilibrium. Volatility of business conditions arises either from fluctuations of the taste shock or fluctuations on the macro level or both. Technically, the assumption that the drift rate of the Brownian motion is given by technological progress instead of an exogenous drift rate of idiosyncratic business condition as e.g. in Bloom (2009) does not influence the mathematical tractability of the model. Nevertheless, it fits the general equilibrium growth perspective of the model in a better way. At each point in time the individual firm chooses the amount of the flexible factor effective labor units $(A_t L_{it})$ to maximize its operating profits $(P_{it}Y_{it} - h(A_t L_{it}))$ where h represents the wage rate per effective unit of labor which is exogenous from the individual perspective of a single firm. The maximized value of operating profit is:[12]

$$\pi_{it}(K_{it}, X_{it}) = \psi h^{1-\gamma\epsilon} X_{it}^{1-\gamma\epsilon} K_{it}^{1-\gamma} \tag{9}$$

$$\psi \equiv (\frac{1}{\gamma\epsilon})^{\gamma\epsilon} (\gamma\epsilon - 1)^{\gamma\epsilon - 1} > 0 \tag{10}$$

$$0 < \frac{1}{\epsilon} < \gamma \equiv \frac{1}{1 + \alpha(\epsilon - 1)} < 1 \tag{11}$$

2.3 Equilibrium in the good sector

Households receive wages and dividend payments from the firm sector which they use either for instantaneous consumption or investment in future consumption. The budget constraint of the representative household at time t is:

$$w_t L + \sum \frac{1}{N} \pi_{it}(K_{it}, X_{it}) = C_t + \sum \frac{1}{N}_{it} \quad for \quad all \quad i = 1, ...; M \tag{12}$$

[11] In the later simulation we will distinguish three different (extreme, but tractable) cases: 1. $\sigma_Z = 0$, $\sigma_A > 0$; 2. $\sigma_Z > 0$, $\sigma_A = 0$; 3. $\sigma_Z = \sigma_A > 0 \Rightarrow \sigma_X = \sigma_Z + \sigma_A$.

[12] More explicit derivation can be found in the single firm partial equilibrium models of Abel and Eberly (1996), Abel and Eberly (1999).

$$I_{it} = \frac{dK_{it}}{dt} \geq 0 \tag{13}$$

where w represents the wage rate. The condition $I_{it} \geq 0$ in eq. (14) reflects the assumption that investment is irreversible.[13] Capital has constant sunk unit cost when installed (q_+). To keep it simple and guarantee a closed form solution, we assume that capital does not depreciate. Nevertheless, we discuss the impact of depreciation later in this paper. The fundamental value of a firm at time t is given by:

$$V_{it}(K_{it}, X_{it}) \equiv max_K E\left\{\int_0^\infty e^{-\rho s}[\pi_{it}(K_{it+s}, X_{it+s})ds - q_+ dK_{it+s}]\right\} \quad with \quad s \geq 0 \tag{14}$$

The equation shows that the instantaneous value of a firm depends on the ongoing maximization in all future periods $t+s$ given the revelation of the shock variable X_{it} and the endogenous reaction of capital adjustment. The cost of this adjustment is shown by the last term on the right hand side. Future output is discounted by the time preference of the households ρ.

Instead of tackling the stochastic capital accumulation directly, we focus on the decision to acquire a marginal unit of capital.[14] This transforms the sequential incremental accumulation problem into an associated simpler optimal timing problem.[15] Since the last term in eq. (11) is not differentiable we follow Bertola (1998) by interpreting it as a Stieltjes integral.[16]

Using the method of dynamic programming[17] the instantaneous marginal value of a single firm can be decomposed into two parts: the marginal value of the actual production and the expected change of the value of the firm. The recursive Bellman equation of the fundamental value of the firms can be written as:

$$V_{itK}(K_{it}, X_{it}) = max_K \left\{\pi_{itK}(K_{it}, X_{it}) + \frac{1}{1+\rho} E_t \left\{V_{it+1K}(K_{it+1}, X_{it+1})\right\}\right\} \tag{15}$$

We define $V(\cdot)$[18] as the expected present value of all future profits under the as-

[13] See (Arrow and Kurz 1970: p.331). Because the marginal contribution of capital is always positive a resale value of zero has the same implication as the total inability to reduce the capital stock.

[14] For an introduction and early application of that method see Pindyck (1988) or Bertola (1998).

[15] (Alvarez 2011: p.1772).

[16] "In the absence of convex installation costs, the rate of growth of capital is unbounded (...)." See (Bertola 1998: p.9).

[17] Because the function will be the same in every time period the time index can be dropped. See (Dixit and Pindyck 1994: p.101).

[18] (Dixit and Pindyck 1994: p.95pp.)

sumption of an optimal investment policy by the single firm. Because of the endogenous reaction of capital adjustment eq. (16) can be simplified to:

$$V_{itK}(X_{it}) = max_K \left\{ \pi_{itK}(K_{it}, X_{it}) + \frac{1}{1+\rho} E_t \left\{ F_{itK}(X_{it+1}) \right\} \right\} \tag{16}$$

Since X_{it} and X_{it+1} could be any of the possible states, we represent them in general form as X_i and $X_i^{'}$. Therefore, for all X_{it} we get:[19]

$$V_{iK}(X_i) = max_K \left\{ \pi_{iK}(K_i, X_i) + \frac{1}{1+\rho} E \left\{ V_{iK}(X_i^{'} \,|X_i, K_i) \right\} \right\} \tag{17}$$

Between two time periods with an interval of Δt we get:

$$V_{iK}(X_i, t) = max_K \left\{ \pi_{iK}(K_i, X_i, t)\Delta t + (1+\rho\Delta t)^{-1} E \left\{ V_{iK}(X_i^{'}, t+\Delta t \,|X_i, K_i) \right\} \right\} \tag{18}$$

Multiplied by $(1+\rho\Delta t)$ and rearranged:

$$\rho \Delta t V_{iK}(X_i, t) = max_K \left\{ \pi_{iK}(K_i, X_i, t)\Delta t(1+\rho\Delta t) + E \left\{ V_{K_i}(X_i^{'}, t+\Delta t) - V_{K_i}(X_i, t) \right\} \right\} \tag{19}$$

$$\rho \Delta t V_K(X, t) = max_K \left\{ \pi_{K_i}(K_i, X_i, t)\Delta t(1+\rho\Delta t) + E \left\{ \Delta V_{K_i} \right\} \right\} \tag{20}$$

Dividing eq. (20) by Δt and letting it go to zero the equation takes the form:

$$_{K_i}(X_i, t) = max_{Ki} \left\{ \pi_{iK_i}(K_i, X_i, t) + \frac{1}{dt} E \left\{ \Delta V_{iK} \right\} \right\} \tag{21}$$

where $\frac{1}{dt} E \left\{ \Delta V_{iK} \right\}$ is the limit of $E \left\{ \Delta V_{iK} \right\} / (\Delta t)$.

[19] Because the function will be the same in every time period the time index can be dropped. See Dixit and Pindyck (1994).

Using Ito's Lemma enables us to derive the expected change in the marginal value as:

$$\frac{1}{dt}E\left\{\Delta V_{iK}\right\} = \mu_A X_i V_{iXK}(K_i, X_i) + \frac{1}{2} \qquad sigma_X^2 X_i^2 V_{iXXK}(K_i, X_i) \tag{22}$$

Together with eq. (21) we get:[20]

$$\rho V_{iK}(K_{it}, X_{it}) = \pi_{iK}(K_{it}, X_{it}) + \mu_A XitV_{iXK}(K_{it}, X_{it}) + \frac{1}{2}\sigma_X^2 Xit^2 V_{iXXK}(K_{it}, X_{it}) \tag{23}$$

$$\rho V_{iK}(K_{it}, X_{it}) = \psi(1-\gamma)h^{1-\gamma\epsilon}\left(\frac{X_{it}}{K_{it}}\right)^{\gamma} + \mu_A XitV_{iXK}(K_{it}, X_{it}) + \frac{1}{2}\sigma_X^2 Xit^2 V_{iXXK}(K_{it}, X_{it}) \tag{24}$$

With eq. (25) we derived the equilibrium condition for capital investment. The households are willing to invest until the marginal benefit of capital at least compensates for the lost opportunity costs. The left side of the equation shows that these opportunity costs are given by the time preference. The right side shows the marginal benefit of investing which can be decomposed into the actual production (the first term on the right side) and the expected change in the fundamental value of the firm on the background of uncertain future demand fluctuations (the last two terms on the right side).[21]

Because the equation is homogenous of degree 1 in both variables (K_{it}, X_{it}), it allows us to normalize the optimization problem by one state variable (K_{it}), and we can rewrite it as $y_{it} \equiv K_{it}/X_{it}$, which is homogenous of degree 0 in both variables.[22] We define the marginal value of capital as:

$$q(y_{it}) = V_{iK}(K_{it}, X_{it}) \tag{25}$$

[20] The equation doesn't contain a term for the cost of capital adjustment. From the condition of optimal adjustment follows that marginal profit is always equal to costs, therefore $dK(V_{iK}(K_{it}, X_{it})) - p_+$ equals zero. If there is no adjustment in capital because if the binding irreversibility constraint, the term will vanish from the marginal examination.

[21] Here, We use the method of dynamic programming ((Dixit and Pindyck 1994: p.104pp.)). Alongside the capital asset pricing model (CAPM) the same result can be shown with the method of contingent claims. Here the expected return will be equal to the expected return of the capital market r, (see e.g. (Abel and Eberly 1996: p.583)) which could equally be interpreted as opportunity costs of investing. Formally the application requires a capital market which our model does not provide.

[22] For the application of this method see Abel and Eberly (1996), Bloom et al. (2007) or (Bloom 2009: p.637).

Again, the value of the Bellman equation can be derived explicitly from Ito's Lemma as:

$$\rho V_{itK}(K_{it}, X_{it}) = \psi(1-\gamma)h^{1-\gamma\epsilon}{y_{it}}^{\gamma} + \frac{1}{dt}E\{dq\} \tag{26}$$

$$with: \quad \frac{1}{dt}E\{dq\} = \mu_A y_{it} q^{'}(y_{it}) + \frac{1}{2}\sigma_X^2 y_{it} q^{''}(y_{it}) \tag{27}$$

$$and: \quad q(y_{it}) = V_{itK}(K_{it}, X_{it}) \tag{28}$$

$$\Rightarrow 0 = \psi(1-\gamma)h^{1-\gamma\epsilon}{y_{it}}^{\gamma} + \mu_A y_{it} q^{'}(y_{it}) + \frac{1}{2}\sigma_X^2 y_{it} q^{''}(y_{it}) - q^{'}(y_{it}) \tag{29}$$

The optimal investment policy can be described in terms of the marginal value of capital $q(y_{it})$. If $q(y_{it})$ surpasses a certain threshold the representative firm will invest until the marginal value equals the threshold again. Formally, if $q(y_{it})$ reaches the investment threshold or boundary the firm acquires a marginal unit of capacity at the cost q_+ and continues operation with a higher capital capacity.[23] The optimal investment policy can therefore be described as a policy controlling the marginal value of capital to be at or under the investment barrier.[24] The optimal threshold itself reflects the Jorgensonian user costs[25] of capital extended by real options:[26]

$$j \equiv \left(1 - \frac{\gamma}{\beta}\right)\rho q_+ \tag{30}$$

$$with: \quad \beta = \frac{1}{2} - \frac{\mu_A}{{\sigma_X}^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu_A}{{\sigma_X}^2}\right)^2 + \frac{2\rho}{{\sigma_X}^2}} < 0 \tag{31}$$

The parameter $1/\beta$ in eq. (30) reflects the option value which increases the investment barrier[27] compared to investment decisions under reversible costs.[28] Here, the user costs of capital would be simply $\rho \cdot q_+$. Optimal investment policy implies that the marginal

[23]See (Alvarez 2011: p.1772).

[24]See (Dixit and Pindyck 1994: p.362).

[25]See Jorgenson (1963). Analogous, it can be compared to Tobin's q. See Alvarez (2011).

[26]For a more extended derivation see Alvarez (2011), Abel and Eberly (1996) and Dumas (1991). For a detailed discussion of the boundary conditions in investment models see Dixit (1989), Dixit (2013). For a general formal derivation see Harrison (1990).

[27]For more details see (Abel and Eberly 1996: p.365pp.(Appendix A.)). Alvarez derives the effect in a more general framework with a broader definition of the production function. See (Alvarez 2011: p.1778).

[28]As Hubbard (1994) states, this can also be expressed in terms of Tobin's q: "(...) the threshold criterion for investment requires that [Tobin's] q exceeds unity by the value of maintaining the option to invest." (Hubbard 1994: p.1819). See also Abel et al. (1996).

value of capital is always at or below the threshold:

$$\psi(1-\gamma)h^{1-\gamma\epsilon}{y_{it}}^{\gamma} \leq \left(1-\frac{\gamma}{\beta}\right)\rho q_{+} \tag{32}$$

With $y_{it} = X_{it}/K_{it}$ and solved for K_{it} the optimal capital stock at the investment boundary is given by:

$$K_{it} = \left[\left(1-\frac{\gamma}{\beta}\right)\frac{\rho q_{+}}{\psi(1-\gamma)h^{1-\gamma\epsilon}}\right]^{-1/\gamma} X_{it} \tag{33}$$

Which means firms' output and profits are given by:

$$Y_{it} = (A_t, L_{it})^{1-\alpha}\left[\left[\left(1-\frac{\gamma}{\beta}\right)\frac{\rho q_{+}}{\psi(1-\gamma)h^{1-\gamma\epsilon}}\right]^{-1/\gamma} X_{it}\right]^{\alpha} \tag{34}$$

$$\pi_{it}(K_{it}, X_{it}) = \psi h^{1-\gamma\epsilon}\left[\left(1-\frac{\gamma}{\beta}\right)\frac{\rho q_{+}}{\psi(1-\gamma)h^{1-\gamma\epsilon}}\right]^{-1/\gamma} X_{it} \tag{35}$$

2.4 Capital accumulation

A first result can be pointed out concerning the impact of real options on the capital stock at the investment barrier. Lifting the irreversibility constraint allows us to compare the case of irreversible investment to the case of reversible investment. Although uncertainty is still prevailing the value of real options in the second case will fall to zero. From eq. (33) it can easily be seen that real options do not have an influence on the growth rate of the capital stock. Denoting variables of the irreversible case with I and the reversible case with R the growth rates are given by:

$$g_{{K_{it}^{*}}^{I}} = g_{{K_{it}^{*}}^{R}} = \frac{E\left(\partial X_{it}/\partial t\right)}{X_{it}} = \mu_A \tag{36}$$

Nevertheless, real options induce a negative level effect on the capital stock at the investment barrier:

$$\frac{{K_{it}^{*}}^{R}}{{K_{it}^{*}}^{I}} = \left[\left(1-\frac{\gamma}{\beta}\right)\right]^{1/\gamma} \tag{37}$$

However, this comparative result does only apply when firms are at their investment barrier in both cases. Under reversible investment costs this will always be the case because firms are not constrained in their capital adjustment policy. Under irreversible investment costs however firms will necessarily find themselves from time to time in a situation where they are constrained by the inability to reduce capital in reaction to a negative realization of the uncertain fluctuations of business conditions. In such a situation the solutions described by eq. (33)-(37) do not apply. Figure 1 illustrates this fact by simulating the evolution of a single firm's irreversible capital stock relative to its counterpart with reversible investment costs. In the latter case, firms are always at their investment barrier.

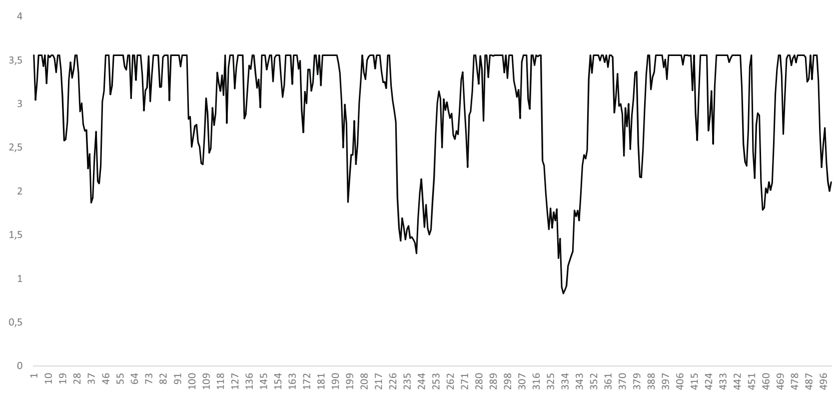

Figure 1: Simulating the evolution of the relative value of the reversible and irreversible capital stock (with $\mu_A = 0,029$ and $\sigma_X = 0,02$).

Figure 1 shows that every time a firm under irreversible investment costs is at its investment barrier the relative value of the two capital stocks reflects the constant multiple described by eq. (37). Because of the real option effect, this multiple is higher than 1. Furthermore, a firm under irreversible costs will constantly face time periods where the irreversibility constraint is binding, leaving the firm with more capital than desired. As the irreversible constraint only prevents capital disinvestment, fluctuations of the capital stock relative to its reversible counterpart are asymmetric. Looking at a single representative firm we would label the values of eq. (33)-(37) as the equilibrium or steady-state values, because the values at the investment barrier are the only stable points in the system. Nevertheless, because of the asymmetric impact of shocks, the expected future "real" value of the capital stock starting from any point in time is always lower than the steady-state value at the investment barrier. Abel and Eberly (1999) show that for a firm born at time 0 (without any capital) and with a normalized idiosyncratic demand process X_{it} to $X_{i0} = 1$ the expected value of the capital stock at any date $t > 0$ can be expressed as:

$$E_0 \left\{ {K_{it}}^R \right\} = \left(\frac{j}{\psi(1-\gamma)h^{1-\gamma\epsilon}} \right)^{-1/\gamma} E \left\{ max_{0 \leq s \leq t} X_{is} \left| X_{i0} = 1 \right. \right\} \tag{38}$$

where the last term reflects the last point of maximization at time $s \leq t$. The expectation term on the right side of the equation can be calculated as:

$$\left[\frac{\mu_A + \frac{1}{2}{\sigma_X}^2}{\mu_A} \phi \left(\frac{\mu_A + \frac{1}{2}{\sigma_X}^2}{{\sigma_X}^2} t^{1/2} \right) e^{\mu_A t} + \frac{\mu_A + \frac{1}{2}{\sigma_X}^2}{\mu_A} \phi \left(\frac{\mu_A + \frac{1}{2}{\sigma_X}^2}{{\sigma_X}^2} t^{1/2} \right) \right] \tag{39}$$

where $\phi(\cdot)$ is the standard normal cumulative density function.[29] With $t \to \infty$ the cumulative density functions will become 1 and 0 respectively so the equation will turn into:

$$\left[1 + \frac{1}{2\mu_A} {\sigma_X}^2 \right] e^{\mu_A t} \tag{40}$$

where $1 < \frac{1}{2\mu_A}{\sigma_X}^2 < 2$ reflects the asymmetric impact of the shocks caused by the irreversible investment costs. Compared to the reversible case this results in a higher expected capital stock under irreversibility at the investment barrier in the long-run. As we will see later in this paper, this will make the use of micro level steady-state values imprecise for the aggregate level. In summary, uncertainty and irreversibility have two effects: First, they lower the level of the capital stock through higher investment barriers caused by real options. Second, the capital stock will be higher because of the asymmetric effects of the binding irreversibility constraint. Whereas both effects are

[29] The complex derivation of this function is based on the definition of $lnX \equiv W$ which changes uncertainty into an arithmetic Brownian motion. Harrison (1990) shows how the above function (32) can be derived with the properties of an arithmetic Brownian motion.

reflected in the expected value of the capital stock, the steady-state value only reflects the first effect because here a firm is at its investment barrier. Even though the simulation in figure 1 clearly indicates that both measures are higher than the reversible capital stock, Abel and Eberly (1999) show that under certain parameter choices for the Brownian motion the second effect can dominate. Figure 2 shows that the ambiguity of the case of Abel and Eberly depends on low levels of uncertainty. With high uncertainty the negative real option effect dominates. The difference between the two curves represents the effect of the binding irreversibility constraint.

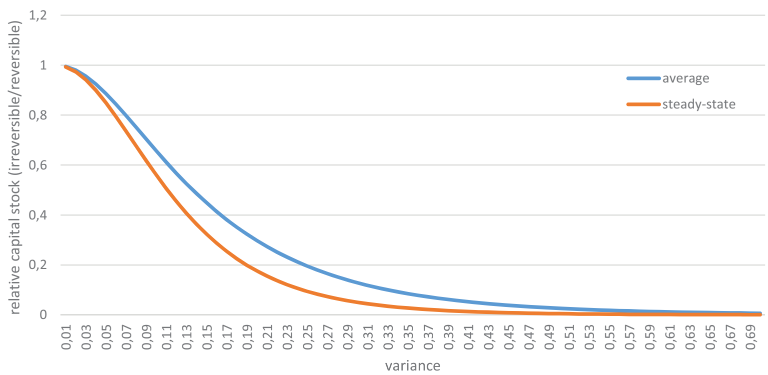

Figure 2: Relative long-run capital stock values as a function of volatility (with $\mu_A = 0,029$).

Concerning the adjustment dynamics of the model, both effects together show how uncertainty and irreversibility induce hysteresis in investment decisions of a single firm over time. The higher investment barrier implies a reluctance of the firm to react to investment incentives on the ”entry side”. On the ”exit side” binding irreversibility delays capital disinvestment.

3 Aggregation

The firm's investment problem described above is highly stylized and of course does not correspond exactly to the complexity of a firm's decision making.[30] Still, our stylized model of capital accumulation yields a closed-form investment rule under reasonable functional form assumptions like former partial equilibrium investment model do [e.g. Bertola (1994)]. Furthermore, it also provides a closed solution for a general setting making real option based investment decisions compatible with a variety of macro models. In general, this enables us to discuss aggregate implications of uncertainty and irreversibility constraints.

On the aggregate level the distinction between idiosyncratic and aggregate uncertainty becomes crucial. So far, we have implicitly assumed that the drift rate of aggregate output $g_Y = \mu_A$ is stable. Since g_Y is the result of aggregate production this implies that the economy at the aggregate level is assumed to be at its equilibrium. Considering long-term growth determinants this assumption might not be problematic, but by looking at dynamic responses to temporary distortions it becomes important. In general, the assumption of uncertainty on the macro level ($\sigma_A > 0$) a source of constant distortions causing recurrent deviations from the economy's equilibrium is introduced.

To show the distinct character of aggregate fluctuations we start first by highlighting the impact of idiosyncratic volatility on the investment behavior of the aggregate firm sector. As we will see, the obtained results do not only serve as a comparison to aggregate volatility, they show interesting implications on their own.

3.1 The impact of idiosyncratic uncertainty

Because of the simplifying assumption that all parameters are the same for every single firm, we can denote aggregate profits, capital and investment as:

$$\Pi_{At} = \sum_{i=1}^{M} \pi_{it}; \quad K_{At} = \sum_{i=1}^{M} K_{it} \quad and \quad I_{At} = \sum_{i=1}^{M} I_{it} \tag{41}$$

Assuming symmetry of relative demand shocks, profits on the aggregate level are independent of the idiosyncratic volatility. A negative demand shock to one firm will be completely offset by a positive demand shock to another firm. On average the shock will

[30] One major problem of the chosen micro-structure to fit real investment series is the absence of zero investment of firms in the data. Bloom (2009)circumvent this problem by assuming that firms consist of a large number of units. The additional "unit-level" in the firm sector leads to smoothing in the investment series. See (Bloom 2009: p.635).

be equal to its mean and therefore vanishes from the aggregate profit function. Consequently, without volatility on the aggregate level the evolution of business conditions will become deterministic following the development of demand generated by aggregate output Y_{At}. Overall profits are given by:

$$\Pi_{At}(K, AL) = \psi h_t^{(1-\gamma\epsilon)} Y_{At}^{\gamma} K_{At}^{1-\gamma} \tag{42}$$

$$with: \quad \psi \equiv \left(\frac{1}{\gamma\epsilon}\right)^{\gamma\epsilon} (\gamma\epsilon - 1)^{\gamma\epsilon - 1} > 0 \tag{43}$$

We can go one step further and explicitly deriving profits by using the production function for the aggregate demand component and the wage level. They are given by:

$$\Pi_{At}(K, AL) = \psi \left[(1-\alpha)\left(\frac{K_{At}}{A_t L_A}\right)^{\alpha}\right]^{(1-\gamma\epsilon)} \left[(A_t L_A)^{1-\alpha} K_{At}^{\alpha}\right]_{At}^{1-\gamma} \tag{44}$$

$$= \psi(1-\alpha)^{(1-\gamma\epsilon)} (A_t L_A)^{-\alpha(1-\gamma\epsilon)-\alpha\gamma+\gamma} K_{At}^{\alpha(1-\gamma\epsilon)+\alpha\gamma+1-\gamma} \tag{45}$$

$$= \psi(1-\alpha)^{(1-\gamma\epsilon)} (A_t L_A)^{1-\alpha} K_{At}^{\alpha} \tag{46}$$

Eq. (46) shows that in the equilibrium profits are always a constant share of output ($\psi(1-\alpha)^{1-\gamma\epsilon}$), the relative strength reflecting the monopolistic power of the firms. Although the idiosyncratic uncertainty vanishes from the aggregate profit function, it still shapes investment decision making of individual firms in the firm sector. Here, the discussed distinction between a firm's steady-state and the expected long run level of the capital stock [eq. (33) and (38)] becomes crucial for the impact of real options on capital accumulation. As described in chapter 2, on the individual level a single firm will go through episodes where it finds itself at the investment barrier, and episodes where it is stuck with more capital than desired. Without aggregate shocks the firm sector will always consist of a fraction of firms which are at their investment barrier and a fraction that suffers from a capital overhang.[31] Since the distribution of the idiosyncratic shock is symmetric the relative size of the two fractions is always constant when the economy is at its equilibrium.

The resulting aggregate capital stock can be derived by using eq. (39). Abel and Eberly (1999) come up with this result for the expected capital stock in the context of a single firm when time and investment decisions approach infinity ($t \to \infty$). We, however, apply

[31] When the economy has not reached its "steady-state" e.g. because of an ongoing catching-up process the balance of user-cost and hang-over effect has not been reached. A balanced growth path can be defined, when the overall capital - efficient labor - ratio is constant.

the properties of the cumulative density functions in eq. (39) to show the implication of firm-specific uncertainty for the aggregate investment level. Letting the numbers of firms M go to infinity the investment decisions in one period following the idiosyncratic shock will approach infinity as well. As a result the first cumulative density function in eq. (39) approaches $\phi(+\infty) = 1$ and the second $\phi(+\infty) = 1$. Since the change in equilibrium is certain we can drop the expectation term. Therefore, starting from a capital stock of 1 at $t = 0$ the equation simplifies to:

$$K_{At} = \left(\frac{\left(1 - \frac{\gamma}{\beta}\right) \rho q_+}{\psi(1-\gamma) h_t^{1-\gamma\epsilon}} \right)^{-1/\gamma} \left[1 + \frac{1}{2\mu_A} \sigma_X{}^2 \right] e^{\mu_A t} \tag{47}$$

Eq. (47) reflects the result that the effect of a binding irreversibility constraint has developed from describing a temporary shock phenomenon at the level of individual firms to describing a constant steady-state level effect at the aggregate level. In contrast to Abel and Eberly's micro perspective, the result at the aggregate level shows that with $M \to \infty$ the long-run investment path is totally stable - defined by the (given) parameters of the model. Again, the equilibrium effect of real options on capital formation depends on the relative strength of entry-hysteresis compared to the effect of irreversibility resulting in firms having more capital than desired as described in chapter 2.4.

Concerning the growth rates in the equilibrium, idiosyncratic volatility does, again, have the same results as at the micro level described in chapter 2. Furthermore, with the aggregation we can now close the model. Because capital grows with the rate of technological progress, we get:

$$\sum_{i=1}^{M} Y_{it} = (A_t \sum_{i=1}^{M} L_{it})^{1-\alpha} \sum_{i=1}^{M} K_{it}^{\alpha} \tag{48}$$

$$= Y_{At} = (A_t L_A)^{1-\alpha} K_{At}^{\alpha} \tag{49}$$

With the growth rate equal to:

$$g_Y = (1-\alpha) g_A + \alpha g_K \quad \Rightarrow \quad g_Y = (1-\alpha)\mu_A + \alpha \mu_A = \mu_A \tag{50}$$

By assuming a constant labor force the wage rate in the equilibrium will also grow with productivity growth: $g_w = \mu_A$. The wage rate in terms of efficient labor will in turn be constant: $g_h = 0$. With profits and output growing at the same equilibrium rate consumption and investment will grow at the same rate as well. In the steady-state the capital stock grows at the equilibrium rate μ_A which is independent of investment. Concerning level effects, the certain long-run level of the aggregate capital stock for the aggregate profits in the firm sector yields:

$$\Pi_{At} = \psi(1-\alpha)^{(1-\gamma\epsilon)}(A_t L_A)^{1-\alpha}\left[\left(\frac{\left(1-\frac{\gamma}{\beta}\right)\rho q_+}{\psi(1-\gamma)h_t^{1-\gamma\epsilon}}\right)^{-1/\gamma}\left[1+\frac{1}{2\mu_A}{\sigma_X}^2\right]e^{\mu_A t}\right]^{\alpha} \tag{51}$$

And for the level of output:

$$Y_{At} = (A_t L_A)^{1-\alpha}\left[\left(\frac{\left(1-\frac{\gamma}{\beta}\right)\rho q_+}{\psi(1-\gamma)h_t^{1-\gamma\epsilon}}\right)^{-1/\gamma}\left[1+\frac{1}{2\mu_A}{\sigma_X}^2\right]e^{\mu_A t}\right]^{\alpha} \tag{52}$$

Again, compared to the case without real options the higher investment barriers reduce profits and output level by a constant factor (j). On the other hand, the effect of binding irreversibility increases profits and output because of the potential higher aggregate capital stock. As in the micro economic perspective, these steady-state values could be higher or lower depending on the model parameters.

3.2 The impact of macroeconomic uncertainty

The different impact of idiosyncratic and aggregate shocks becomes apparent at the aggregate level. From the micro perspective of the representative firm the two kinds of shocks have a common feature. Through β both types of uncertainty have the same implications on investment (entry) decisions. Furthermore, they induce the same amount of potential capital hang overs. The single firm does not distinguish between these two forms of uncertainty when deciding about the amount of capital investment.

However, ex-post the impact will be different, because of the homogenous effect on all firms. A deviation from the expected mean induces a dynamic investment and output reaction. If one suppose e.g. a positive, higher than expected shock to A_t at the aggregate level the aggregate capital stock would be too low to guarantee a stable capital coefficient. The relative scarcity of capital will induce a higher profit rate and higher aggregate investment. In addition, since efficient labor is abundant, the wage rate of efficient labor will fall and therefore increase the investment incentive further. Because of the adjustment process the growth rate of aggregate output will be higher than μ_A. This leads to an additional dynamic response of economy-wide investment, output and growth under irreversible investment costs. The change in g_{Y_t} will influence the real option parameter β itself. In case of an unanticipated shock to productivity the higher drift rate will temporarily lower the option value of investing thereby easing the hysteresis effect.

Neglecting the feedback effects on real options on the aggregate level, as models like Bloom et al. (2012) do, leads to an overestimation of the quantitative impact of real options in a general equilibrium setting. This is especially true if autoregressive shocks to volatility are introduced like in Bloom (2009). In case of a positive shock to volatility real option values would increase. But since there is no change in the growth perspective of the economy the lower degree of investment would raise the marginal profitability of capital which would in turn decrease the option of waiting.

3.3 Summary of findings

In sum, the results show that by extending micro decision making and partial equilibrium models, basic findings on real options are still valid in a more general setting. As eq. (30) shows, real option values lead to a higher investment barrier for firms in the firm sector, causing a reluctance to react to investment incentives. Together with a binding irreversibility constraint this causes the familiar hysteresis effect in investment and disinvestment. By highlighting these two effects our results confirm the micro economic findings of Abel and Eberly (1999) in that the influence of uncertainty on the long-term capital stock is not per se clear (eq. (38)). With respect to growth theory, our model confirms theoretical findings that the long-term growth rate (μ_A) remains independent of investment when a real option based micro-foundation is added. Here, our predictions are in line with the results of Jones (1995), Blomström et al. (1996) and Attanasio et al. (2000), who state that there is no clear evidence for a link between investment and growth.

Next, our results provide new insights concerning the dynamic effects on aggregate investment and capital accumulation in a general equilibrium setting. In particular, unlike Bloom et al. (2012), the results of this paper show that different kinds of uncertainty have different effects on adjustment dynamics. Even though both types of volatility shape investment decisions of firms in the same way, idiosyncratic volatility vanishes on the aggregate level if the number of firms approaches infinity. Aggregate volatility, on the other hand, does not only cause fluctuations on aggregate investment series, it also has an influence on the short-term growth dynamics by pushing the economy out of its equilibrium. A positive (negative) shock causes the marginal capital value to be higher (lower) compared to the equilibrium rate. The temporary higher (lower) drift rate of business condition leads to temporary lower (higher) real options values. This effect also reduces the effect of volatility shocks on real options.

In general, by endogenizing the drift rate of business conditions in a growth model, our paper provides the next step in the investigation of potential dynamic effects of real option values. Earlier research on investment in micro and macro models has treated real option values as constant and exogenous. Bloom (2009) has introduced shocks to volatility to generate fluctuations in real option values to discuss uncertainty as a driving force of investment cycles. Although his findings offer important new insights on the uncertainty-investment-link his model only generates semi-endogenous real options. The variance of those values depends only on volatility which is in turn an exogenous process. Our model in contrast focuses on the determination of the long-term drift rate of business conditions thereby connecting them to growth issues. In addition, the model can easily be extended by exogenous shocks to volatility in the lines of Bloom by (partly) endogenizing both variables of real option values.

The discrimination of the effects of different types of volatility also has consequences for the classification in terms of growth terminology. At the level of a single firm's capital stock the hysteresis effects of idiosyncratic volatility together with irreversibility can be classified as an equilibrium level effect (user cost effect) at the entry side and temporary disturbances caused by a binding irreversible constraint at pöthe exit side. At the aggregate level, however, idiosyncratic volatility is completely ”washed out”. The two effects together now describe a constant level-effect for aggregate investment (eq. (47)). Concerning aggregate volatility, the classification of the two effects in terms of growth theory as level effect and temporary disturbance applies on both the firm's and the aggregate capital stock level.

4 Simulation and Future Research

To highlight the effect of volatility and real options on investment and growth patterns we will run a simulation of the model with a firm sector consisting of 2000 firms[32] in the first part of this chapter. The simulation first shows the different impact of the two different sources of volatility on fluctuations of investment series. Next, we will show the dynamic impact of shocks on our simulated economy. To highlight the effect of real

[32]For the moment the firm sector is clearly not big enough to completely wash out idiosyncratic volatility completely. See (Bloom 2009: p.643) for a discussion of the magnitude of the firm sector to match aggregate investment time-series.

options we will compare the case of investment under irreversibility with its reversible counterpart. In the second and third part we will discuss implications of this model for future research work (4.2) and potential extensions (4.3).

4.1 Simulation of investment series with different sources of volatility

In general, the complexity of working with a two-dimensional random process limits us to discuss three tractable cases. In the first case fluctuations are purely idiosyncratic with $\sigma_Z > 0 \quad and \quad \sigma_A = 0; \quad so \quad \sigma_X = \sigma_Z$. In the second case fluctuations steam completely from aggregate fluctuations with $\sigma_Z = 0 \quad and \quad \sigma_A > 0; \quad so \quad \sigma_X = \sigma_A$. Here, investment pattern of the firm sector follow the more dynamic impact described in section 3.3. In the last case we assume that both distortions have the same volatility and distribution. We set $\sigma_Z = \sigma_A > 0 \quad so \quad \sigma_X = \sigma_Z + \sigma_A > 0$. As has been shown by Bloom et al. (2012), Hatzius et al. (2012) or Balta et al. (2013) the last case reflects reality best because different measures of volatility are closely correlated.

Nevertheless, to show the different impact of different sources of volatility we initially compare the first two cases. We set the drift rate to 4 per cent, capital share to one third, demand elasticity to 10, capital cost to 40, the time preference to 0,05 and the labor force to 1000. Idiosyncratic and aggregate volatility is set to 0,1 respectively.

Figure 3 and 4 show how the different cases of volatility influence the aggregate investment series. Although the value of volatility is chosen relatively high, figure 3 shows that the impact of idiosyncratic volatility is close to zero. In fact, it would be completely down to zero if the number of firms in the firm sector approached infinity.[33] Aggregate volatility in turn translate directly into a more volatile investment series.

4.2 Simulation of the investment response to different shocks to volatility

To highlight the dynamics of investment in the light of real options we will now show how the model reacts to different kinds of shocks. We will follow Bloom (2009) to show the effect of a pure shock to volatility. As Bloom states, such a shock can be identified with e.g. a strong rise in uncertainty after major economic and political events.[34] We model a temporary shock to volatility by rising it to 0,3. After the impact the shock fades out with an autoregressive coefficient of 0,9.

[33] The impact of idiosyncratic volatility on aggregate investment series depending on the number of firms in the firm sector highlights the fact that the number of firms has a diversification externality for the households. The higher the investment opportunities the stronger the effect of absorbing idiosyncratic volatility. See Acemoglu and Zilibotti (1997) for a theoretical discussion of this effect.

[34] See (Bloom 2009: p.673).

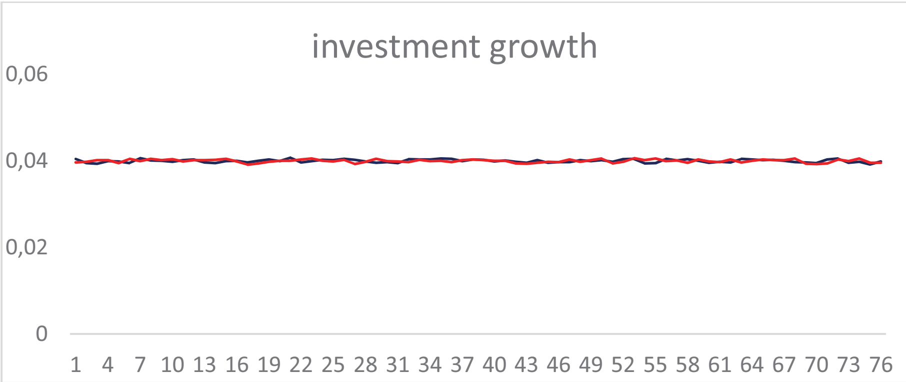

Figure 3: Aggregate investment growth with pure idiosyncratic volatility.

Figure 5 shows the effect on the two investment time series. Leaving the reversible case (red) nearly unaltered,[35] the impact only affects the irreversible investment series by causing strong dynamics in real options. As in Bloom, the pattern describes a drop in investment at the impact of the volatility shock. This highlights the effect of a rise in uncertainty leading to a ”wait-and-see” attitude of firms. The temporary stop in investment causes a pent-up investment demand in the mid-turn. Therefore, investment overshoots before falling back to the long-run growth rate.

As a second shock we induce a negative aggregate supply shock to technology. Without changing any of the other variables, we assume that the economy suffers from a once-and-for-all drop of technology of 15 per cent to highlight the dynamics of the rebalancing process of the economy. The growth rate of technology is kept at 4 per cent.

The investment series in figure 6 show how real options, through their hysteresis effect, change the dynamic rebalancing process after a first-moment shock. On the one hand, reversible investment (blue) enables the firms to get back on the investment barrier

[35] One can observe a slightly stronger fluctuation in the investment series of the reversible case (blue) which is again the consequence of a finite number of firms in the firm sector. So, the stronger volatility can’t be absorbed completely.

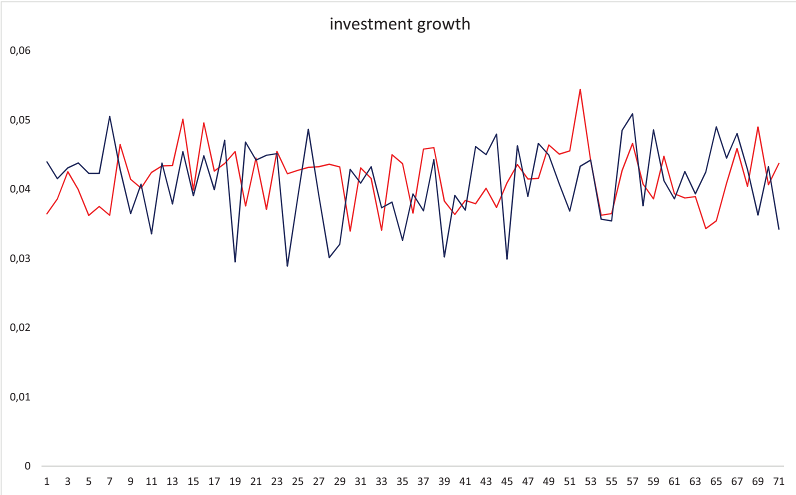

Figure 4: Aggregate investment growth with pure aggregate volatility.

at once through disinvestment. On the other hand irreversible capital investment (red) delays a rebalance of the capital stock. Overall firm's investment goes to zero, followed by a phase of slow recovery which follows from the hysteresis effect of binding irreversibility.

4.3 Future research

In general, the consequences of uncertainty and volatility on investment and capital accumulation shown in the model offer new insights for future research. First, as has been shown by Bloom (2009) and Bloom et al. (2012) macroeconomic models with a real options micro-foundation potentially offer important and new implications for short-run investment dynamics. Bloom et al. (2012) connect business fluctuations to shocks to volatility, thereby offering a first step to endogenize real option values. Nevertheless, because the shocks to volatility are themselves exogenous the induced change in real option values can only be labeled as semi-endogenous. By linking the drift rate of business condition to the state of the overall economy our model offers richer dynamics which can be applied in more sophisticated models.

Recently, the implication of changing real option values has gained some attention in the discussion on the driving forces of low investment in the aftermath of the financial

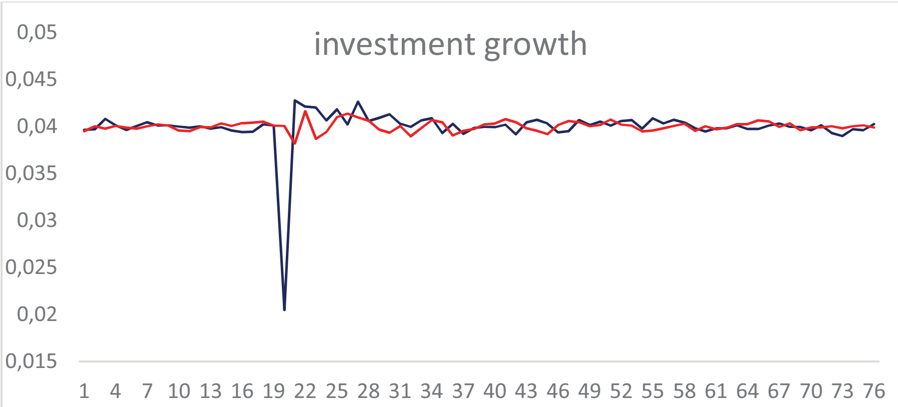

Figure 5: Aggregate investment dynamics following a second-moment shock to idiosyncratic volatility.

crisis and the euro crisis. In that respect Baker et al. (2013) and Bloom and Floetotto (2009) have pointed out that potentially uncertainty, especially policy uncertainty, in the US is a major driver for weak investment. A similar case has been made by Balta et al. (2013) and Buti and Mohl (2014) for the Euro area. Because our model incorporates a richer setting in terms of feedback effects of volatility on the aggregate level, it potentially alters the quantitative predictions of models like Bloom et al. (2012). In particular, our results suggest that the hysteresis effect after a positive shock to aggregate volatility is overestimated when output is below its potential. Future research could clarify how significant these effects are.

Second, our model offers a way to introduce real option effects in the field of development economics. In that respect, our model offers two new theoretical channels showing the potential impact of uncertainty on capital accumulation and catching-up dynamics.[36] On the one hand, our model shows that real options lower the willingness to take advantage of investment possibilities. As a consequence the speed of convergence in a catching-up process is reduced. In equilibrium the hysteresis effect plays an important

[36]"When growth first starts, it is driven by capital accumulation,(...)" (Aghion et al. 2009: p.226).

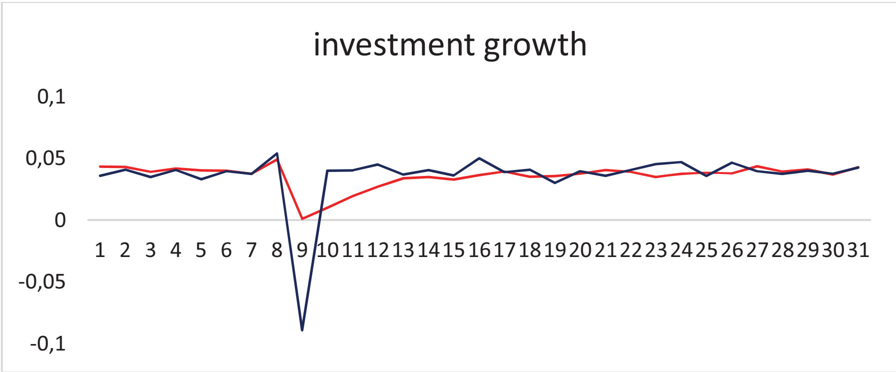

Figure 6: Aggregate investment dynamics following a first-moment technology shock.

role on both investment and disinvestment.[37] Nevertheless, if the capital stock is beneath its equilibrium level investment hysteresis becomes asymmetrical. As Abel and Eberly (1999) state "(...) irreversibility reduces the expected value of the initial capital stock [starting from no capital] because only the user-cost effect is operative for the initial capital stock; the hangover effect is inoperative because the firm has not yet accumulated any capital in the past." [38] As we have shown in our general equilibrium model, the effect of a higher investment barrier will dominate the investment process in the beginning however leaving the long-run growth rate unaffected.[39] On the other hand, because of the scarcity of capital our model predicts the marginal value of capital and therefore the drift rate of business conditions to be higher so option values will be lower at low levels of capital accumulation. This effect therefore weakens the effect of the dominating user cost effect. It remains an open question which effect dominates and,

[37]See the critique of (Bloom 2000: p.17).

[38](Abel and Eberly 1999: p.349).

[39]A variety of partial equilibrium models with two periods [e.g. Caballero (1991), Pindyck (1993), Sakellaris (1994) and Lee and Shin (2000)] generates an inverse relationship between uncertainty and investment just by assuming that firms start with no capital. Such an assumption can easily be justified in the context of development economics. See Bloom (2000) for a discussion of these models.

more general, if real options are an important feature in investment driven catching-up dynamics.

4.4 Extensions

The model is kept simple to show the basic insights in a closed-form solution. It can be augmented by a variety of extensions, none of them changing the qualitative results found above.

First, one could lift the assumption of total irreversibility. Alvarez (2011) and Abel and Eberly (1996) show that the coefficient of the relative value of the investment barrier compared to the reversible investment case is positive and increasing with respect to the degree of irreversibility in costs. Therefore, the result will be qualitatively the same.[40] For future research in the field of real business cycles it would be interesting to show how aggregate fluctuations are also reflected in the degree of irreversibility. As Abel et al. (1996) state, irreversibility is likely to be high when potential buyers suffer from the same shock that resulted in the firm's decision to sell capital in the first place.[41] Endogenizing irreversibility would therefore strengthen the results of this paper concerning the discrimination between aggregate and idiosyncratic shocks even further.

Second, also adding a positive depreciation rate would not alter the qualitative results. In general, depreciation will have the same implication as a higher drift rate in a firm's investment decisions. The constraint of binding irreversibility would be less painful because waiting will more quickly reduce the overhang in capital for firms with a binding irreversibility constraint.

Third, Alvarez (2011) numerically shows that changing the random process into a mean reverting process would not change the qualitative findings of this model.[42] Again, it would lower the impact of uncertainty on investment as well. Concerning the underlying uncertainty process, unit level uncertainty like in Bloom et al. (2012) can be added. Although the empirical evidence for this sub-firm level is less convincing [43] unit level uncertainty smooths the investment path of firms thereby providing a better prediction of actual investment data. With respect to our model, such an extension would have the advantage to lower the high sensitivity of aggregate investment to the level of volatility.

[40] For a more complex derivation, with partial irreversibility see (Abel and Eberly 1996: p.587), proposition 4. Also the derivation of the irreversible investment path in (Alvarez 2011: p.1773; p.1778) Theorem 3.1 and 3.2.

[41] (Abel et al. 1996: p.755).

[42] Because of the mathematical complexity a closed solution can't be derived. With estimated parameters Alvarez (2011) show that the qualitative implications stay the same.

[43] See Bloom (2009).

In general, the framework offers a new way to discuss and investigate different sources of uncertainty, e.g. uncertainty about costs (Bertola (1994)), uncertainty about the real interest rate (Ingersoll and Ross (1992)) or policy uncertainty (Hassett and Metcalf (1999), Pawlina and Kort (2005) and Baker et al. (2013)). The implications concerning the general hysteresis effect are qualitatively the same. Wherever there is uncertainty combined with irreversibility real options emerge causing hysteresis in decision-making. Nevertheless, our results underpin the importance on which level the particular uncertainty does emerge. An overall higher dynamic of taste shifts won't necessarily alter aggregate fluctuations and influence the drift rate of business cycles.

5 Conclusion

Uncertainty and irreversibility have a significant impact on investment decision-making. This paper shows how real option based investment decisions can be integrated into a simple general equilibrium model. The structural framework we have developed in this paper consists of a micro-founded investment sector which faces two kinds of uncertainty at the firm and the aggregate level. By using this in a simple Ramsey-style growth model we extend the real option literature in several ways. First, we generalize capital accumulation under real options by looking at a sector of firms instead of a single firm. Second, we connect investment decisions with household's utility thereby extending the model in order to make growth predictions. Third, we add different sources of volatility that affect firm's decision making simultaneously. Forth, by identifying the growth rate with the drift-rate of business conditions our framework endogenize real option values. And finally, by using a simulation we show the dynamic reaction to aggregate shocks.

Our findings show that basic findings of earlier research can be generalized by placing them in a general equilibrium setting. In this respect, our findings support the results of partial equilibrium models in that the effect of uncertainty on the level of the steady-state capital stock depends on parameter choice and in that the hysteresis effect shown in the real options literature does prevail also on the aggregate level. With respect to growth theory our model supports the view that the long-run growth rate of the steady-state capital stock is independent of investment and therefore also independent of volatility.

In addition, our results show that different kinds of volatility have different implications when moving from a partial to a general equilibrium model. Idiosyncratic uncertainty vanishes in the aggregation process, but still shapes the adjustment dynamics of the model due to the micro structure of investing firms. Therefore, our model predicts that overall higher idiosyncratic fluctuations don't cause aggregate fluctuation if the number of firms in the firm sector approaches infinity. An interesting aspect concerning idiosyncratic volatility is that in the aggregation process the long-run capital stock of a single firm converges to a stable growth path without fluctuations. Although idiosyncratic uncertainty prevails in the decision making of the firms the steady-state capital stock and the growth rate of the economy converge to a predictable value in the long-run.

As far as aggregate volatility is concerned the model predicts that different types of volatility incorporates different effect on aggregate investment behavior. In particular, our model highlights the effect of aggregate shocks on real option values themselves. We show that the assumption of a constant drift rate is only true if the economy is at its steady-state when aggregate business conditions depend on the state of the overall economy. Outside the equilibrium the change in business condition and the marginal value of capital are not stable. Because the value of real options depends both on volatility and the drift rate of business condition they also incorporate a dynamic effect in the aftermath of aggregate shocks leading to temporary diversions from the equilibrium.

In general, by investigating the endogeneity of real options in neoclassical growth models our paper provides a single framework to shed light on both long-run growth and level effects as well as temporary adjustment dynamics. Therefore, the framework can serve as a basis for a variety of future research. The endogenous real option values can be used to further investigate the role of real option dynamics in real business cycles, following Bloom (2009) and Bloom et al. (2012). Furthermore, the growth model approach of this paper offers a starting point to use real option based macro models also in related research fields like development economics. Here, the effect of real options on investment dynamics potentially plays an important role in catching-up processes. It thereby offers a deeper understanding of the effect of uncertainty on economic growth.

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