University of Freiburg
Department of International Economic Policy
Discussion Paper Series
Nr. 20

Monetary and Fiscal Policy in a Monetary Union under the Zero Lower Bound constraint

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May 2012

ISSN 1866-4113

University of Freiburg
Department of International Economic Policy
Discussion Paper Series

The Discussion Papers are edited by:
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Editor:
Prof. Dr. Günther G. Schulze

ISSN: *1866-4113*
Electronically published: 29.06.2012

Monetary and Fiscal Policy in a Monetary Union under the Zero Lower Bound constraint

Stefanie Flotho*

May 24, 2012

Abstract

This paper explicitly models strategic interaction between two independent national fiscal authorities and a single central bank in a simple New Keynesian model of a monetary union. Monetary policy is constrained by the zero lower bound on nominal interest rates. Coordination of fiscal policies does not always lead to the best welfare effects. It depends on the nature of the shocks whether governments prefer to coordinate or not coordinate. The size of the government multipliers depend on the combination of the intraunion competitiveness parameters. They get larger in case of implementation lags of fiscal policy.

JEL-Classification: E31, E52, E58, E61, E62, E63, F33

Keywords: Monetary Union, Fiscal Policy, Zero Lower Bound on nominal interest rates, zero interest rate policy, Non-coordination

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1 Introduction

Since the 1990s, Japan has a short rate very close to zero. Because of the Great Recession of 2008–2009 central banks have set interest rates below one percent in the Euro Area, the United States, England and Switzerland, among others. Thus, as monetary policy is restricted by the nominal interest rate channel it is not able to boost the economy and stop the economic downturn. So, fiscal policy is called to help. After the financial crisis, several governments have implemented major fiscal stimulus packages in 2008 and 2009. However, in practice, all of these packages differ in size, and it is not clear which instruments (e.g., tax cuts, government spending, or deficits) help to stimulate the economy. Even in the academic debate there is no consensus in the size of the fiscal effect. Whereas some theoretical analysis on fiscal multipliers conclude that these are large in a liquidity trap, other studies take a different view.

Research has spent a whole agenda on studying and explaining optimal monetary policy, comparing optimal interest rules with Taylor rules and analyzing the interaction between monetary and fiscal policy. Although sometimes contradictory, the results are well understood and influence central banking. However, all of the wisdom becomes obsolete when monetary policy is constrained by the zero lower bound (ZLB) on nominal interest rates. When an economy moves toward a liquidity trap with negative output and inflation gaps, and a binding constraint on the nominal interest rate, central bankers have a hard time stabilizing the economy.

The ZLB constraint changes the problem under consideration both economically and computationally. The introduction of the ZLB constraint makes the optimization problem of the central bank nonlinear with a kink at zero in the nominal interest rate. As a result, the analytical solution method for rational expectations models of Blanchard and Kahn (1980) is not longer applicable.

However, not only computational methods have to be adjusted: the focus of the economical reasoning also has to change. For example, in a standard, dynamic New Keynesian model under normal circumstances, without a binding ZLB constraint, demand shocks play a less significant role, because optimal monetary policy and Taylor rules can offset a positive demand shock in a trivial way by raising the nominal interest rate with no effect on any other economic variable. Under the ZLB constraint, however, this result does not hold any longer. Fiscal policy is implemented, but how effective are these efforts?

Moreover, because several countries are hit by the ZLB at the same time, it has to be explored whether coordination of fiscal policies across countries might stop an economic recession and help to terminate the time with zero nominal interest rates faster than without coordination.

This article contributes to both questions. Using a simple two-country New Keynesian model of a monetary union, government spending multipliers when monetary policy is optimal or follows a Taylor rule are computed and compared with the values of the multipliers when the nominal interest rate is zero. The size of the various multipliers depends on the combination of the intraunion competitiveness parameters and is not necessarily large when the ZLB constraint binds. However, the fiscal effects are amplified when there are implementation lags in government spending.

The second question is posed slightly differently. The focus changes when considering the length of the ZLB to be endogenous or exogenous: When the time when ZLB ends is endogeneous, then the question to be answered is: "Which policy can terminate this policy?" If it is assumed that the length is exogenous, the question is "How does fiscal policy react to shocks while monetary policy is not able to set its instrument?" The latter question is answered. It is analyzed how the coordination of both fiscal policies or the simultaneous setting of country-specific government spending can stabilize several different shocks. It depends on the nature of the shocks whether governments prefer to coordinate or not coordinate.

Research on fiscal multipliers in a monetary union and dynamic effects of coordinated or uncoordinated fiscal policies under a ZLB is still scarce. Thus, this paper adds to recent research about fiscal stimulus packages as it computes fiscal multipliers in a currency union. With a focus on intraunion competitiveness spillovers it complements and extends Cook and Devereux (2011) and Fujiwara and Ueda (2010), which analyze the issue in a setup with flexible exchange rates.

The next section gives a short literature review. Section 3 introduces the theoretical framework of the monetary union. In Section 4 the strategic interactions of the benchmark case of full coordination of both governments and the Nash game of simultaneous setting of policy instruments are analyzed and compared. Impulse response functions to several shocks and welfare effects are analyzed. Section 5 computes different multipliers. The final section concludes.

2 Literature Review

With the beginning of the Great Recession the field of fiscal policy has gained renewed interest. For the past two or three years, articles published on macroeconomic policies in a liquidity trap can be divided into two parts. The first strand deals with the analysis of fiscal multipliers.[1]

[1]This paper focuses on theoretical optimizing models. For examples of the empirical analysis of fiscal multipliers, see the articles by Blanchard and Perotti (2002), Mountford and Uhlig (2009), Romer and Romer (2010), Ilzetzki et al. (2010) or Traum and Yang (2011). All articles find estimates of government spending multipliers by close to one on impact. However, depending on the data set and identification scheme values

In contrast, the second strand analysis the dynamic effects of fiscal policy while the nominal interest rate is constrained by the zero lower bound.

After the financial turmoil and the practical implementation of fiscal stimulus packages, research is trying to determine the effects of fiscal policy calculating fiscal multipliers. However, instead of giving a single coherent answer to the question how big the multipliers are theoretical analysis has lead to a "fiscal multiplier morass" (Leeper, 2010, p.19). In the academic debate, there is no consensus in the size of the fiscal effect. In standard New Keynesian models, government spending multipliers are usually below or near one at impact. Whereas some theoretical analysis on fiscal multipliers conclude that these are large in a liquidity trap (Eggertsson, 2010; Christiano et al., 2009; Woodford, 2011),[2] other studies take a different view (Cogan et al., 2010).[3]

More recent examples for the analysis of fiscal multipliers in a closed economy with or without a ZLB constraint are Romer and Bernstein (2009), which seems to have kicked off the whole controversy. Christiano et al. (2009), Davig and Leeper (2011), Woodford (2011), Eggertsson (2010), or Erceg and Lindé (2010) implement the framework of New Keynesian models, whereas Uhlig (2010) calculates multipliers in a neoclassical growth model. To overcome the many different conclusions about the size of the fiscal multipliers Cogan et al. (2010) and Coenen et al. (2012) compute and compare multipliers across a wide class of models. However, their studies lead to opposing conclusions.[4]
All of these models can be extended in various manners. For example, to model the recent financial crisis more explicitly, many researchers include financial frictions using the financial accelerator mechanism of Bernanke et al. (1999) into the basic framework of a New Keynesian model. They investigate the role of monetary and fiscal policy in the presence of such frictions (Carrillo and Poilly, 2010a,b).

In the rest of this section, I focus on optimizing New Keynesian models with a focus on two or more countries or regions that analyze the effects of multipliers and fiscal spillovers.

Fujiwara and Ueda (2010) is close to the analysis of this article. However, the authors consider

are estimated in a wide broader range and can even become negative.

[2]This group defends their analytical result with the argument that fiscal policy is more effective when the nominal interest is bounded at zero. When the central bank cannot counteract the fiscal expansion and increasing inflation rates by increasing nominal interest rates they do not crowd out consumption or investment.

[3]These authors, for example, use a model database of different macro models to investigate the effects of the *American Recovery and Reinvestment Plan*, and conclude that the stimulus is much smaller than predicted by Romer and Bernstein (2009) when analyzed in New Keynesian type models. In a companion article Cwik and Wieland (2011) perform the same analysis with regard to the European fiscal stimulus packages: Their results suggest smaller multipliers.

[4]One reason for the difference might lie in the definition of the multiplier. The latter focus more on long-run multipliers in contrast to one-period impact multipliers.

an optimizing sticky price model of two countries that do not form a monetary union but set their monetary policies independently from each other. They compare analytical government spending multipliers in normal times with those that arise when both countries are caught in a liquidity trap. The multipliers are smaller when central banks can react to fiscal expansion by setting the nominal interest rate. In their model, it depends on the size of the intertemporal elasticity of substitution of consumption whether there are negative fiscal spillovers on the neighbor country.

Using a similar setup, namely a standard new open economy macroeconomic model of two independent countries, Cook and Devereux (2011) compare government spending multipliers in the case when one or both countries are caught in the liquidity trap with the case when monetary policy operates under a Taylor rule. Fiscal policy expansions can help to stimulate the domestic economies, but there are significant negative cross country spillovers via the terms of trade. Moreover, they find that the multipliers are even magnified in open economies in contrast with closed economies because in their setup, the exchange rate depreciates.[5]

Simulating a large-scale DSGE model, the Euro Area and Global Economy model,[6] Gomes et al. (2010a) obtain numerical values for fiscal policy multipliers in the ZLB. Their results suggest that multipliers are higher when the ZLB constraint binds than in normal times. The difference between multipliers in both scenarios is higher for the United States than for the Euro Area. As a next step, they find that the fiscal stimulus package implemented by the United States even helps to overcome the periods with a binding ZLB, but the fiscal stimulus package of the Euro Area is not sufficient to end the ZLB constraint.

Similar results are obtained by Coenen et al. (2012). In this meta-study of seven structural models, of which one is a two-region model and four are global models, the authors conclude that fiscal policy multipliers are enlarged when the nominal interest rates are constant. This result is valid across all the different models.

Cwik and Wieland (2011) is another study comparing the effect of the fiscal stimulus packages of the Euro Area across several structural models. However, as indicated earlier, they conclude that the size of the multipliers does not increase under the ZLB. In contrast with the aforementioned article, these authors focus on long-run fiscal multipliers and analyze a permanent government spending shock.

Another part of the literature that does not calculate multipliers examines the effects of fiscal policy when the ZLB holds with the help of impulse response functions. To date, to my

[5]This is in contrast with open economy results that imply that exchange rates appreciate under a fiscal expansion and thus multipliers are dampened by a fall in net exports.

[6]This model consists of four blocs, namely Germany, the rest of the Euro Area, the United States and the rest of the world (Gomes et al., 2010b).

knowledge only a handful of papers has performed this exercise within the framework of New Keynesian models of multiple countries.

In addition to the aforementioned result about the fiscal multipliers, Cook and Devereux (2011) observe that the joint coordination of both fiscal policies is not necessarily optimal in a liquidity trap. Instead, there should be a greater fiscal expansion in the country that is more hit by a liquidity trap than in the neighboring country. They only focus on the optimal policy mix of joint fiscal policies, but do not consider other strategic interactions.

Gomes et al. (2010a) analyze which policy instruments can terminate the duration the ZLB constraint holds. Thus, it is assumed that the time length of the ZLB constraint is given endogenously. The authors simulate a severe global recession hitting the four blocs (Germany and the rest of the Euro Area, the United States, and the rest of the world). As a result, nominal interest rates which are set via a Taylor rule are driven to zero in all countries. Then, they compare the effectiveness of several temporary fiscal shocks to reduce the duration of the ZLB constraint. Throughout their simulation, the authors assume that fiscal policy is credible and sustainable all the time. Government consumption (in contrast with government transfers) is the most powerful tool in all countries. In addition, they find that fiscal spillovers play a minor role in their model.

In contrast, **?** find very sizeable fiscal cross country spillovers in a two-region model of a monetary union. They analyze the effects of asymmetric shocks in a currency union where the central bank usually sets the nominal interest rate via a Taylor rule, but is constrained by the ZLB for some periods. Fiscal policies in both regions (calibrated to the North and the South of the European Union) are independent. Governments can issue nominal debt to finance their deficits.

3 The Model

In this article the monetary union is modeled to consist of two countries, H(ome) and F(oreign). Both economies, $j = H, F$, can be summarized by a forward-looking New Keynsian Phillips curve and a demand curve:

$$\pi_t^j = \beta E_t \pi_{t+1}^j + \lambda x_t^j + u_t^j \tag{1}$$

$$x_t^j = E_t x_{t+1}^j - \varphi(\bar{\iota}_t - E_t \pi_{t+1}^j) + g_t^j + \gamma \left(x_t^i - x_t^j \right) - \delta(\pi_t^j - \pi_t^i) + \varepsilon_t^j \quad i \neq j,\, i = H, F \tag{2}$$

All variables should be read as deviations from their respective values at an efficient steady state. Inflation dynamics in both countries are driven by forward-looking elements in the form of conditional expectations of the future domestic inflation rate $E_t\pi^j_{t+1}$ for $j = H, F$, the domestic output gap x^j_t, $j = H, F$, and a country-specific cost-push shock u^j_t, $j = H, F$. The discount rate β and the output gap coefficient λ (the slope of the Phillips curve) are both positive. The coefficients of the Phillips curve are equal for both countries. All assumptions are imposed to keep the model as simple as possible.

County-specific demand is related inversely to the real interest rate which is the difference of the nominal interest rate $\bar{\imath}$ and the expected future country-specific inflation rate. The nominal interest rate is set by the monetary policy and is the same for both countries. So, monetary policy affects output directly through the interest rate channel. Real interest rates differ as a result of possible different inflation expectations. Changes in domestic demand as a result of spillover effects in the monetary union are modeled by the term $\gamma(x^j_t - x^i_t)$, for country $i = H, F$, $i \neq j$. If $\gamma > 0$, an increase in the output gap abroad leads to an increase of the domestic output gap. However, if $\gamma < 0$, then an increase in the demand of the neighbor country leads to a reduction of the home demand. Moreover, inflation differentials are included to account for intraunion competitiveness channels as in Andersen (2008) and Michalak et al. (2009). ε^j_t, $t = H, F$, are country-specific demand shocks other than fiscal policy g^j_t, $j = H, F$, which increases demand directly. Both shocks ε^j_t and u^j_t follow $AR(1)$-processes.

The increase of government spending is like a demand shock in the economy. Higher spending in the home country raises demand for the goods sold in the home country. As a consequence, the monopolistically competitive firms in the home country increase their demand for labor, which results in higher real wages and higher marginal costs. Those firms that can set new prices according to the Calvo contracts increase their prices leading to higher inflation. In sum, a positive domestic demand shock in form of government spending leads to higher domestic inflation rates and higher domestic product regardless of the design of fiscal or monetary policy. Via the intraunion competitiveness channels, the increase in domestic government spending has an effect on the foreign economy. Depending on the size of the spillover effects, either positive or negative cross-country multipliers can be observed. This is discussed in great detail in Section 5.

The focus of the paper is to analyze how monetary and fiscal policy interact when the economy is constraint by the ZLB. Thus, it is not explicitly modeled which kind of shock has pushed the monetary union into the situation of zero nominal interest rates. However, all of these possible shocks (e.g., natural interest rate shock, discount factor shock, or shock to the risk premium) can be summarized by the demand shock ε^j_t if necessary.

4 The Different Scenarios Under the ZLB

In this section, the different policy scenarios of possible interaction between monetary and fiscal policies are analyzed under the additional assumption that the nominal interest rate is equal to zero all the time. The length of the ZLB constraint is assumed to be exogenous. Thus, the aim of this section is a descriptive explanation of how the endogenous variables react to a shock while the ZLB constraint holds. To find out which combination of country-specific fiscal policy terminates the time length of zero short-term policy rates is left for future research.

The case of full coordination of fiscal and monetary policy is compared with the strategic interactions of simultaneous setting of all three instruments. Given that the interest rate is equal to zero, policymakers can only set fiscal instruments. So, the benchmark scenario is equivalent to the case of coordinating home and foreign fiscal policy. The regime of monetary leadership is constrained by the rule that the nominal interest rate is zero. Fiscal leadership coincides with the regime of the Nash game. The focus is on discretionary fiscal policy.

In the case of a *Nash game of full noncoordination*, the two governments set country-specific government spending g_t^j, $j = H, F$ to minimize their country-specific loss functions which depend on national output gaps and deviations of governmental spending:

$$\min_{g_t^j} L_t^j = \frac{1}{2}\left((x_t^j)^2 + \theta(g_t^j)^2\right). \tag{3}$$

They are not concerned about national inflation. Similar loss functions including government spending can be found in Uhlig (2003) or Andersen (2008).

Optimal responses of the governments are identical for both countries.

$$g_t^j = -\frac{1}{\theta}\frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}x_t^j, \qquad j = H, F.$$

Aggregating and taking differences yield

$$\begin{aligned} g_t^W &= -\frac{1}{\theta}\frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}x_t^W \\ g_t^R &= -\frac{1}{\theta}\frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}x_t^R \end{aligned} \tag{4}$$

In contrast, the common monetary authority focuses on aggregate variables of the monetary union and thus includes the aggregate inflation rate π_t^W and the aggregate output gap x_t^W in the central bank's loss function. The instrument is the nominal interest rate.

$$\min_{\bar{\iota}_t} L_t^M = \frac{1}{2}\left(\alpha(x_t^W)^2 + (\pi_t^W)^2\right), \tag{5}$$

where $\alpha > 0$ denotes the relative weight of inflation over output.

The constraint of the central bank is given by the aggregated Phillips curve, the demand curve and the nonlinear constraint $\bar{\imath} \geq 0$. To see the technical problem that arises under the ZLB, note that the first-order conditions of this nonlinear optimization problem can be summarized by the Kuhn-Tucker conditions[7]

$$i_t > 0 \quad \Longrightarrow \quad \alpha x_t^W + \lambda \pi_t^W = 0 \tag{6}$$

$$i_t = 0 \quad \Longrightarrow \quad \alpha x_t^W + \lambda \pi_t^W \leq 0 \tag{7}$$

In the *benchmark case of full coordination*, policymakers cannot set the nominal interest rate $\bar{\imath}$, but they choose aggregate government spending g_t^W and relative government spending g_t^R to minimize the loss function, which is a weighted average of the fiscal and monetary loss functions (3) and (5).[8]

$$\min_{g_t^W, g_t^R} \frac{1}{2} \left((1+\alpha)(x_t^W)^2 + (\pi_t^W)^2 + n(1-n)(x_t^R)^2 + \theta (g_t^W)^2 + n(1-n)\theta (g_t^R)^2 \right), \tag{9}$$

where α, θ are positive constants denoting the relative weight of the output gap, and government spending.

Constraints of this optimization problem are given by the aggregate demand and Phillips curves taking into account that the nominal interest rate $\bar{\imath} = 0$. First-order conditions with respect to aggregate government spending g_t^W, and relative government spending g_t^R are given by

$$\begin{aligned} 0 &= (1+\alpha)x_t^W + \lambda \pi_t^W + \theta g_t^W \\ 0 &= (1+\alpha)\frac{(\gamma + \delta\lambda)(2n-1)}{1+2\gamma+2\delta\lambda} x_t^W + \frac{\lambda(\gamma+\delta\lambda)(2n-1)}{1+2\gamma+2\delta\lambda}\pi_t^W + \frac{n(1-n)}{1+2\gamma+2\delta\lambda} x_t^R + n(1-n)\theta g_t^R \end{aligned}$$

Combining these two equations yields

$$0 = -\theta \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} g_t^W + \frac{n(1-n)}{1+2\gamma+2\delta\lambda} x_t^R + n(1-n)\theta g_t^R \tag{10}$$

[7] These conditions are obtained by setting up the Lagrangian of the problem attaching the three constraints of the problem with three Lagrangian multipliers, taking derivatives with respect to the inflation rate, output gap and the nominal interest rate, and combining all three first-order conditions.

[8] The resulting loss function is given by

$$L_t^M + nL_t^H + (1-n)L_t^F \tag{8}$$

In the next step, I assume that the monetary union is in a period when the ZLB constraint binds: that is, monetary policy cannot react to any disturbances. During this time both countries are hit by various cost-push or demand shocks. I analyze how the governments can react to these shocks, and how the reaction is different from the situation in normal times when the central bank can set the nominal interest rate.[9]

To visualize the dynamic effects of shocks on the different variables the model is calibrated as in table 1. The values assigned to the different parameters are comparable to other simulation studies in related literature: The discount factor β is set to 0.99. Following Galí and Monacelli (2008), the interest rate elasticity of output φ is set to be equal to 0.75.[10] The slope of the Phillips curves λ is set to be equal to 0.25.[11] The parameters measuring competitiveness effects in the demand equations are set to $\gamma = 0.5$ and $\delta = 0.5$, which is in line with the papers by van Aarle et al. (2004) and Michalak et al. (2009). The monetary policy sets the relative weight in the central bank's loss function to 0.5. In contrast, governments want to stabilize output and put more weight on output stabilization ($\theta = 0.1$). Shocks are assumed to be highly persistent with a coefficient ρ^j of 0.9.

Parameter		Value
Discount factor	β	0.99
Interest rate elasticity	φ	0.75
Slope of the Phillips curve	λ	0.25
Trade spillovers	γ	0.5
Price competitiveness	δ	0.5
Relative weight in the loss function of the central bank	α	0.5
Relative weight in the loss function of governments	θ	0.1
Home country size	n	0.75
AR term of cost-push shock	ρ_u	0.9
AR term of demand shock	ρ_ε	0.9

Table 1: Calibration

Figures 1 and 2 show the impulse response functions of the policy instruments and the country-

[9] The latter case is analyzed in detail in a similar model to the one in this paper in Flotho (2012).

[10] Estimates for this parameter vary from 0.4 for the US (McCallum, 2001) and an average value of 0.7 for the EU, ranging from 0.4 in Portugal to 1.2 in Germany (Cecchetti et al., 2002). The value chosen here is also in line with van Aarle et al. (2004).

[11] Usually, as shown in Galí and Monacelli (2008), λ is a parameter depending on the structural equations of an underlying microfounded model. Taking the standard values for a closed economy the slope of the Phillips curve can be computed to be equal to 0.0425. Taking into account a microfounded model of a small open economy of a monetary union, Galí and Monacelli (2008) compute a value of 0.3718. Other calibration of this value range from 0.1 for the US (Rotemberg and Woodford, 1999) to 0.3 (McCallum and Nelson, 1999). The value chosen in this paper is in line with Herz et al. (2006) and van Aarle et al. (2004).

specific inflation rates and output gaps to a *symmetric demand shock* both for the regime when the ZLB constraint binds and when it does not bind. Policy instruments are set under the benchmark regime of full coordination.

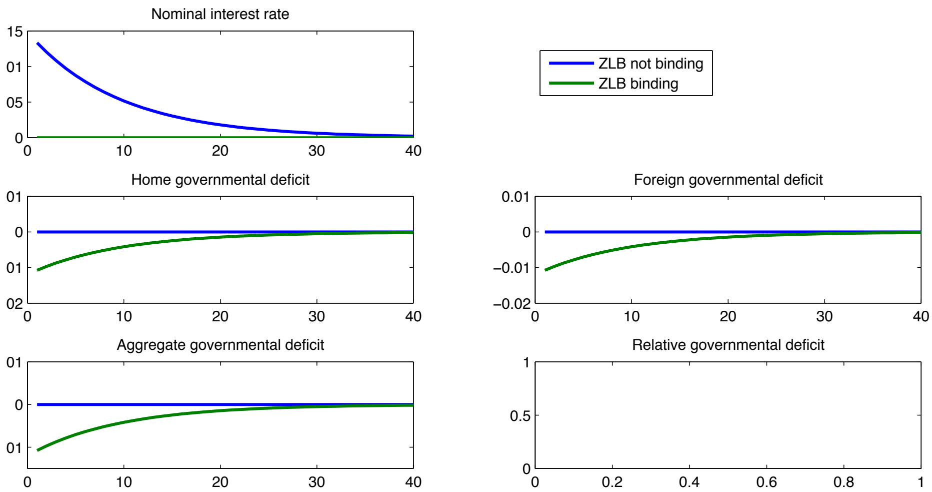

Figure 1: Impulse responses of policy instruments to a symmetric demand shock under a binding and not-binding ZLB constraint

In normal times fiscal policy does not react at all when the monetary union (which is assumed to be symmetric in country size) is hit by symmetric demand shocks. In this case, the central bank can solely offset the shock by increasing the nominal interest rate. Neither output gaps nor inflation rates respond to the shocks. Obviously, this result does not hold any longer under the policy regime when the ZLB constraint binds. Monetary policy cannot stabilize the demand shock. Fiscal policy has to be set. A positive aggregated demand shocks leads to an immediate increase of the aggregate output gap and via the inflation dynamics equation to an increase in the aggregate inflation rate. The first optimality condition of the benchmark scenario holds if aggregate government spending decreases. Given that there is no relative demand shock, the real output gap differential x_t^R does not change. Assuming that both countries are of the same size, the optimality condition (10) implies that governments set their spending such that relative government spending g_t^R does not change. As a result, country-specific output gaps and government spending change in the same direction and of the same magnitude. The decrease of the fiscal policy instruments dampens the inflationary boom in both countries and on aggregate.

If both countries are of different size, both governments have to coordinate their policies such that the optimal response (10) holds. So, they might set their policy instruments having different sizes implying nonzero relative government spending. This influences the relative

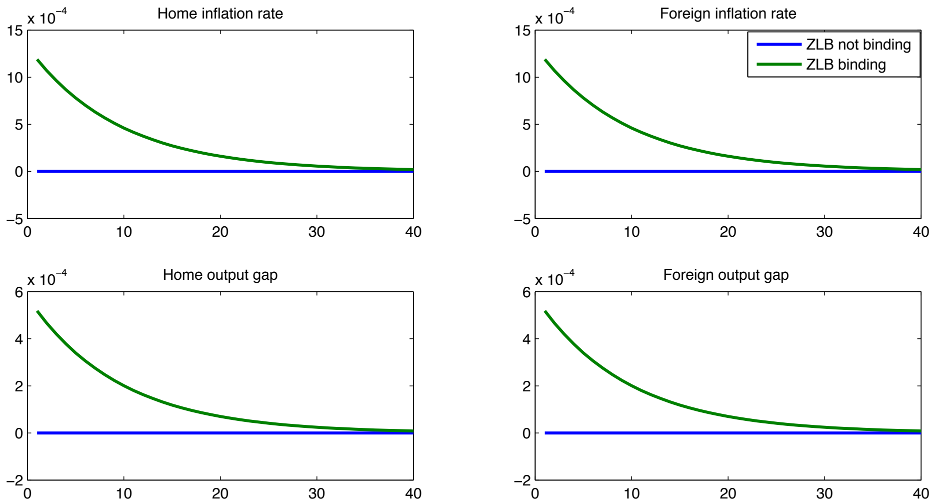

Figure 2: Impulse responses of inflation rates and output gaps to a symmetric demand shock under a binding and not-binding ZLB constraint

output gap and because of spillover effects country-specific variables respond differently to the symmetric demand shock.[12]

If governments do not coordinate but set their instruments simultaneously, they decrease government spending to stabilize the inflationary output booms in both countries. They react symmetrically no matter of which country size they are according to the optimality conditions (4) of the Nash game which do not depend on n.

To illustrate further that fiscal policy is different in the case that monetary policy is not capable to react to shocks than in normal times the case of *symmetric cost-push shocks* is analyzed. Figures 3 and 4 show the impulse response function for the benchmark case of full coordination both when the ZLB constraint binds and does not bind.

The positive symmetric cost-push shocks immediately increase the country-specific inflation rates and thus the aggregate inflation rate. To satisfy the first-order conditions of the benchmark scenario, aggregate government spending has to decrease. Assuming equal country-size changes of both home and foreign fiscal policies are the same in direction and size. This contracting fiscal policy leads to a recession in both countries. Inflation rates decrease, and the shock is stabilized in both countries.

In contrast, when the ZLB does not bind, the symmetric cost-push shock is fully stabilized by an increase of the nominal interest rate. In the case of full coordination, the usual trade-off

[12] The influence of spillover effects is discussed in great detail in Section 5 about multipliers.

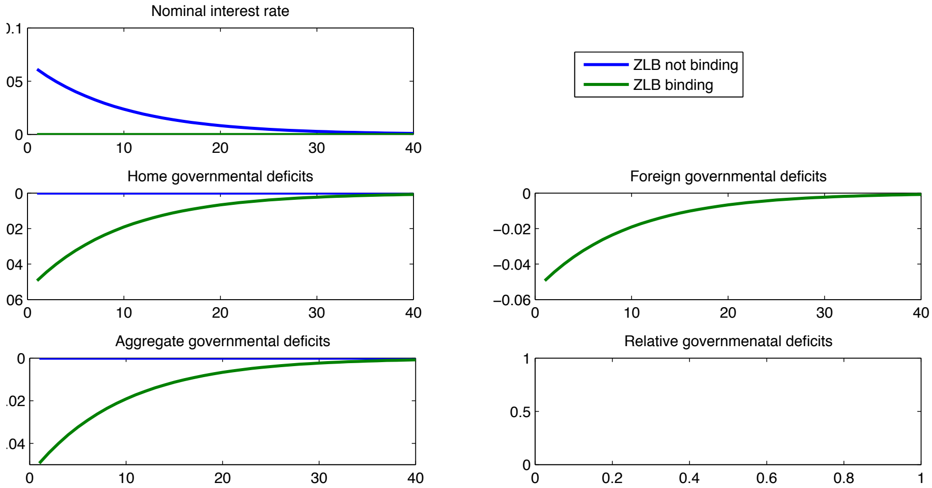

Figure 3: Impulse responses of policy instruments to a symmetric cost-push shock under a binding and not-binding ZLB constraint

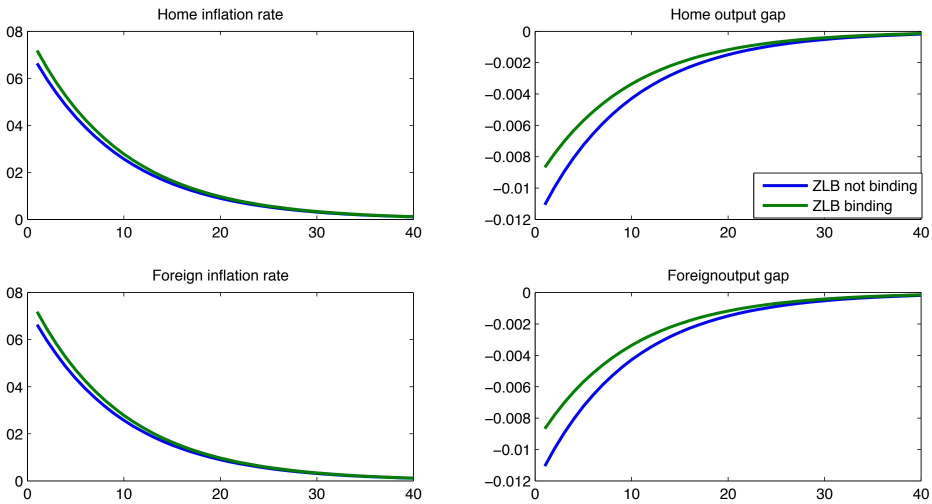

Figure 4: Impulse responses of inflation rates and output gaps to a symmetric cost-push shock under a binding and not-binding ZLB constraint

of monetary policy between stabilizing the output gap and the inflation rate holds. Fiscal policy does not react to a symmetric cost-push shock.

When policymakers set their instruments simultaneously, the reaction to a symmetric cost-push shock are similar to that in the benchmark case. The responses do not differ in direction, but in size. Under noncoordination all reaction are stronger than in the benchmark case and they do not depend on the country-size.

In sum, the reaction of fiscal policy under the ZLB constraint is different from the reaction in normal times.

The question that has to be explored further is which kind of strategic interaction is the best for the monetary union under the ZLB constraint, i.e. whether home and foreign fiscal policies should be coordinated or set simultaneously. So, in the rest of the section, I analyze how the different policy regimes differ in case of asymmetry in shocks. More precisely, the cases of a single home cost-push and home demand shock are analyzed.

However, to continue, the calibration has to be adjusted to the new computation. As for the former calibration, the case of the Nash policy regime yields indeterminacy the parameter δ is changed to $\delta = -1/2$.[13]

The different effects of a home cost-push shock are given in figure 5. In the benchmark case the effects of the home variables are similar to those effects of a symmetric cost-push shock, as a similar transmission mechanism of the shock holds. Differences arise because first of all, just one country is hit by the shock, whereas the foreign country reacts when spillovers lead to a recession in the foreign country. Second, spillover effects are different in this case as calibration has been changed.

Setting instruments simultaneously, the responses are different. The cost-push shock leads to mild booms in both countries and the reaction of the fiscal policy instruments change.
As a result of the high negative spillovers from the home to the foreign country, the response of the foreign government spending in both policy regimes is higher than the response of the home fiscal policy.

Concerning welfare, at first sight, it is not obvious which strategic interaction governments prefer. Ranking both policy regimes with the help of the loss functions results in the following conclusion. In case of a home cost-push shock, both countries prefer to coordinate their

[13]Eigenvalues of the system under noncoordinated policies do not satisfy the Blanchard-Khan conditions when calibrating the model as in Table 1. Note that one has to look at the eigenvalues of the matrix $(Id(4) - B_1 D_1)^{-1} A_1$, where $Id(4)$ is the $4 \times 4-$identy matrix, the matrices A_1 and B_1 are spelled out in Appendix F, and D_1 is a 2×4-matrix giving the reaction of the country-specific governmental instruments to the endogenous variables, that is, in the case of the Nash regime the fiscal policy reaction functions:

$$D_1 = \begin{pmatrix} -\frac{1}{\theta(1+2\gamma+2\delta\lambda)} & 0 & 0 & 0 \\ 0 & -\frac{1}{\theta(1+2\gamma+2\delta\lambda)} & 0 & 0 \end{pmatrix}$$

$(Id(4) - B_1 D_1)^{-1} A_1$ depends highly nonlinear on the parameters of the model. Algebraic expressions of all the matrices and the eigenvalues are not reported here. However, they are calculated with the help of the program Mathematica 7, and the files are available upon request from the author.

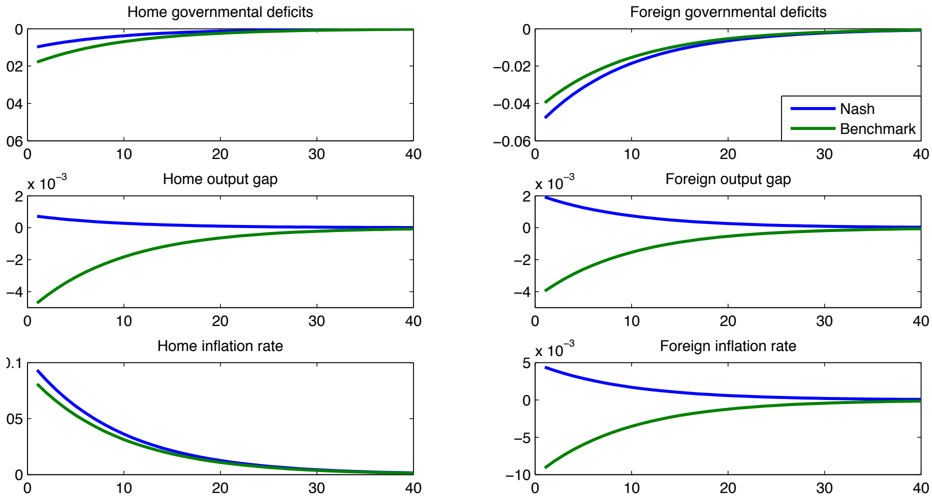

Figure 5: Comparison of Benchmark and Nash I: Impulse responses to a home cost-push shock

policies. For the foreign country, the difference of the losses under coordination and the losses under Nash is bigger than for the home country. So, the foreign government prefers more to coordinate than to set instruments simultaneously. Table 6 of Appendix C reports values of the loss functions for the given calibration.

Figure 6 analyzes the reaction of the model to a home demand shock under the two regimes of full coordination and noncoordination.[14] The demand shock leads to an immediate increase of the home output gap, and thus government spending has to decrease. When coordinating both policies under the assumption of equal country-size, both fiscal instruments have to decrease. As a result, there is a boom in the foreign country. In contrast, when setting instruments simultaneously just the home government reacts to the boom according to the fiscal optimality condition.

In the benchmark scenario, all responses are greater than in the Nash scenario. So, it is obvious that in the case of a single home demand shock both policymakers prefer not to coordinate their policy actions, but to set instruments simultaneously. The welfare effects are compared in Table 6 of the Appendix. The difference of the losses of the benchmark scenario and the Nash scenario is much bigger for the foreign country than for the home country. According to the numbers the home country is almost indifferent between both

[14]The impulse response function for the home fiscal instruments in case of the Nash regime almost coincides with the reaction in the benchmark scenario. The reactions of the foreign variables in case of the Nash game are almost zero.

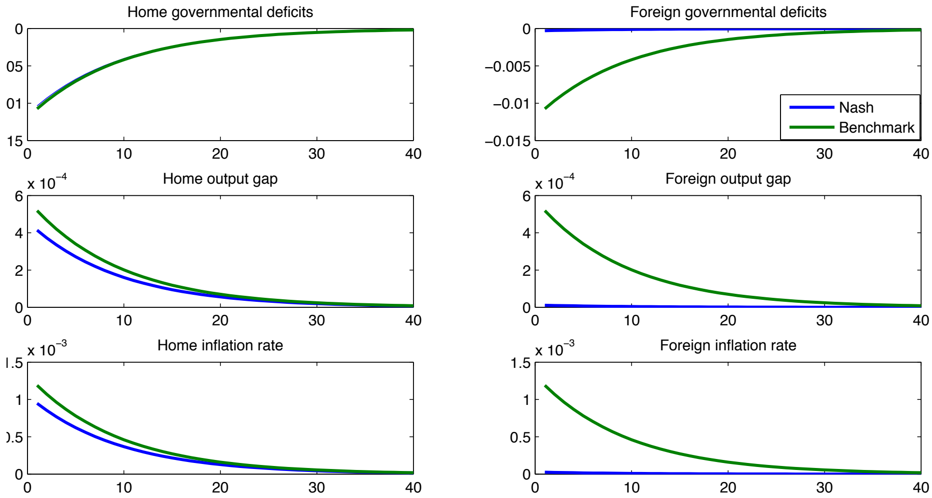

Figure 6: Comparison of Benchmark and Nash II: Impulse responses to a home demand shock

policy interactions.

To conclude, it depends on the nature of the shocks whether governments prefer to coordinate or not coordinate.

5 The Multiplier

To analyze the effect of fiscal policy, various multipliers are analyzed in this section. Because the model in this paper abstracts from explicit modeling of tax policy and spending policy, from now on the variable g_t is interpreted to be government spending (rather than deficits).

Fiscal policy under different assumptions is considered. First, I assume that the central bank can freely adjust the nominal interest rate, that is, multipliers in normal times are derived. To get results that are comparable to other studies, I make the additional assumption that in this case the central bank follows a Taylor rule. Moreover, the case of optimal monetary policy is discussed. Second, these are compared with fiscal multipliers when the nominal interest rate is zero, that is, fiscal policy is the single stabilization tool.
When analyzing the effects of fiscal stimulus packages in both countries, the focus is to find an answer how the monetary union at a whole is affected and how these effects translate to the single countries. Thereby intraunion spillover effects and the cross-country transmission of

fiscal policy play an important role. Last, the question is answered how large the multipliers are.

The linear rational expectations model of Section 3 is given in the following form

$$X_t \;=\; AE_tX_{t+1} + BG_t + \varepsilon_t,$$

where $X_t = (x_t^H, x_t^F, \pi_t^H, \pi_t^F)$ is a vector of the endogenous variables, $G_t = (i_t, g_t^H, g_t^F)$ is the vector of the policy instruments and $\varepsilon_t = (\varepsilon_t^H, \varepsilon_t^F)$ is a vector of the shocks. The matrices of coefficients that are not necessarily diagonal are given by A and B.[16]

Solving this equation forward for periods, $j = t, t+1, \ldots, T$ leads to

$$X_t \;=\; A^T E_t X_{t+T} + \sum_{j=0}^{T-1} A^j B E_t G_{t+j} + \sum_{j=0}^{T-1} A^j E_t \varepsilon_{t+j},$$

Thus, the vector X_t of domestic and foreign output gaps and inflation rates depends on three terms. First, the at time t expected values of these variables at time $t+T$. Imposing the assumption, that the constraint on the ZLB holds until time $T-1$ and that the monetary policy can act in periods following time T as in normal times, expectations about the outcome of the interaction of monetary and fiscal policy play a role.Second, the home and foreign variables depend on expectations of future policy variables. Moreover, given that the matrices are not diagonal, spillover effects can be seen here. So, changes in domestic policy has an effect on variables abroad. Third, the last term on the right hand side captures the effect of expected future shocks. To summarize, expectations strongly amplify the effects of shocks or policy setting on home and foreign output gaps and inflation variables.

The immediate effect of a change in the policy instruments dG_t on the output gaps and inflation rates is given by the one-period multiplier, which is analyzed in this section:

$$\frac{dX_t}{dG_t} = B. \tag{11}$$

The one-period impact on the domestic and foreign output gaps are given by

$$dx_t^j \;=\; \left[-\varphi d\bar{\iota}_t + \frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda} dg_t^j + \frac{\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda} dg_t^i\right] \; j = H, F \; i \neq j. \tag{12}$$

Referring to aggregate and relative variables, the multipliers of the aggregate and relative output gaps are given by

$$dx_t^W \;=\; \left[-\varphi d\bar{\iota}_t + dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} dg_t^R\right] \tag{13}$$

$$dx_t^R \;=\; \frac{1}{1+2\gamma+2\delta\lambda} dg_t^R \tag{14}$$

[16] See Appendix A for the definition of the matrices A and B.

Immediately several conclusions from these mathematical expressions can be drawn.
First, when the ZLB constraint does not bind, the increase of output as a result of fiscal stimulus is less than that in the case when the ZLB binds. A positive effect of an increase in government spending is dampened by an increase of the nominal interest rate, as the central bank wants to maintain a stable inflation rate.

Second, when the fiscal stimulus in both countries are of the same size, then the multiplier on the output gap is equal to $(1-\varphi)$ in normal times, and equal to 1 when the zero lower bound constraint binds. Spillover effects do not play any role when both government agree on the same size of change of government spending. This results of a very small multiplier is in line with many other publications discussed in the literature review.

Third, the aggregate government spending multiplier of the aggregate output gap always is equal to 1. The multiplier of relative government spending depends on the country size and intraunion spillovers. The higher the size of the home country, the higher the multiplier in absolute value. The relative government spending multiplier of the relative output gap is less than 1 in absolute value and does not depend on the change of the nominal interest rate.

Forth, the multipliers get larger under the assumption of implementation lags in fiscal policy. This issue is discussed in more detail in Section 5.4.

To analyze the effects of government spending, different assumptions on how the central bank sets the nominal interest are imposed. The next section starts with the analysis of optimal monetary policy, then monetary policy sets the policy instrument following a Taylor rule. In Section 5.3 the nominal interest rate is bound at zero.

5.1 The Multiplier When Monetary Policy Is Optimal

The optimal decision of the central bank leads to the optimality condition (6) $\pi_t^W = -\alpha/\lambda x_t^W$, where α is the weight in the loss function denoted to inflation stabilization.
In the absence of aggregate cost-push shocks, there is no trade-off between stabilizing the aggregate output gap and the aggregate inflation rate. The central bank can close all gaps, that is, optimal monetary policy implies that $\pi_t^W = x_t^W = 0$. Inserting this into the aggregate demand equation gives the following reaction of the nominal interest rate

$$\begin{aligned} \bar{\iota} &= \frac{1}{\varphi}\left(g_t^W + (2n-1)\gamma x_t^R + (2n-1)\delta\pi_t^R + \varepsilon_t^W\right) \\ \Rightarrow \qquad d\bar{\iota} &= \frac{1}{\varphi}\left(dg_t^W + (2n-1)\gamma dx_t^R + (2n-1)\delta d\pi_t^R\right) \end{aligned}$$

The differential of the relative inflation rate and output gap are given by (see (19) of the appendix)

$$d\pi_t^R = \frac{\lambda}{1+2\gamma+2\delta\lambda} dg_t^R$$

$$dx_t^R = \frac{1}{1+2\gamma+2\delta\lambda} dg_t^R$$

These relations lead to the following multipliers:

$$dx_t^W = 0$$

$$dx_t^H = dx_t^W - (1-n)dx_t^R = \frac{-(1-n)}{1+2\gamma+2\delta\lambda} dg_t^R$$

$$dx_t^F = dx_t^W + n dx_t^R = \frac{n}{1+2\gamma+2\delta\lambda} dg_t^R$$

The first result is obtained by implementing the differential of the nominal interest rate into (13). For the second and the third result it is used that country-specific variables can be expressed as a combination of aggregate and relative variables.[17]

The government spending multipliers in the case of optimal monetary policy are relatively simple. All aggregate government spending multipliers are zero. The central bank sets the nominal interest rate to offset aggregate disturbances. So, an aggregate fiscal policy does not serve as a stabilization tool.
Cross-country multipliers are symmetric. Moreover, relative spending multipliers depend on the country size n, and the competitiveness parameters γ and δ. The effect of home government spending on the home output gap is positive if the sum $(1 + 2\gamma + 2\delta\lambda)$ is positive, that is, if $(\gamma + \delta\lambda) > -1/2$. Moreover, the multiplier of foreign government spending of the home output gap is of the same size, but negative. The home government spending multiplier is infinite if the sum $(\gamma+\delta\lambda) = -1/2$. The cross-country multiplier then is minus infinity.

To illustrate the cross-country transmission of government spending assume, for example, that both competitiveness parameters are negative and do not satisfy the aforementioned inequality. An increase in the home government spending at impact leads to a positive output gap in the home country by one-to-one. The home inflation rate increases via the Phillips curve mechanism. As trade spillovers are negative, the foreign output gap becomes negative which dampens via the real differentials x_t^R the boom in the home country, but improves the conditions in the foreign country at the same time. As the nominal differential π_t^R is negative,

[17]It is clear that the same results are obtained by substituting the aforementioned differentials into (12) and (??).

the boom of the home country is dampened even more and the output gap even becomes negative, whereas nominal spillovers to the foreign country lead to a boom in the neighbor country.

As a result an increase in home government spending might lead to a recession in the home country, whereas the neighbor country profits from the stimulus because of trade spillovers.

The size of the cross-country multipliers depend on the combination of the competitiveness parameters. Table 2 gives numerical values for various combinations of the parameters.[18]

	$\gamma = 1/2$	$\delta = 1/2$			$\gamma = -1/2$	$\delta = 1/2$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	0	-0.11	0.11	-0.11	0	-1	1	-1
dx_t^F	0	0.33	-0.33	0.33	0	3	-3	3
dx_t^W	0	0	0	0	0	0	0	0
dx_t^R	0	0.44	-0.44	0.44	0	4	-4	4
	$\gamma = -1/2$	$\delta = -1/2$			$\gamma = 0$	$\delta = 0$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	0	1	-1	1	0	-0.25	0.25	-0.25
dx_t^F	0	-3	3	-3	0	0.75	-0.75	0.75
dx_t^W	0	0	0	0	0	0	0	0
dx_t^R	0	-4	4	-4	0	1	-1	1

Table 2: Numerical multipliers when monetary policy is optimal

5.2 The Multiplier With a Taylor Rule

In this section, the central bank follows a Taylor rule when setting the nominal interest rate. This implies that output gaps are not necessarily closed, and inflation rates differ from their targets. To simplify computations the Taylor rule is given by

$$\bar{\imath} = \gamma_\pi \pi_t^W + \gamma_x x_t^W$$

where the coefficients $\gamma_x > 0$ and $\gamma_\pi > 1$ to follow the Taylor principle, and the target inflation rate is set to zero. Under this rule

$$d\bar{\imath} = \gamma_\pi d\pi_t^W + \gamma_x dx_t^W$$

Inserting this equation first into (13), then the total differentials for $d\pi_t^W$ and dx_t^W as given

[18]The case describes above is an example for the second row, first column of the table.

in the Appendix A into the result and finally solving for dx_t^W and $d\pi_t^W$ yields

$$dx_t^W = \frac{1}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi}dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^R$$

$$d\pi_t^W = \frac{\lambda}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi}dg_t^W + \frac{(2n-1)\lambda(\gamma+\delta\lambda)}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^R$$

When the expansion of aggregate government spending is positive and first assuming that relative government spending does not change, that is both governments decide to increase their fiscal instrument by the same amount, then the aggregate output gap increases. An aggregate fiscal stimulus has a direct positive effect on aggregate demand that is one-to-one (this can be seen when aggregating the home and the foreign demand curves). The aggregate output gap becomes positive leading to an increase in the aggregate inflation rate via the aggregate Phillips curve. Both increases imply that monetary policy raises the nominal interest rate according to the Taylor rule. This dampens aggregate demand. Thus, the size of the aggregate fiscal multiplier is less than 1.

Second, assuming that both governments raise their fiscal spending, but the increase of the foreign governmental instrument is larger than the one of the home country. In this case, relative government spending is positive and affects intraunion competitiveness and the stimulus depends on the spillover parameters γ and δ, and the country size n of the home country. The larger all these parameters the larger is the effect on the aggregate variables.[19] Moreover, the effect depends on the sign of the parameters γ and δ, and whether the home country is more than one half in size of the monetary union. Assuming that both parameters γ indicating trade spillovers and δ indicating price competitiveness are positive, a positive increase in relative government spending first raises real divergences x_t^R among both countries by less than one. As a direct consequence, nominal divergences measured by the relative inflation rate π_t^R increases. If the home country size $n > 1/2$, aggregate demand increases and implies a positive aggregate output gap with the same consequences as described in the first case. If the home country is smaller than the foreign country, then a positive change in relative government spending dampens aggregate demand.

To analyze the intraunion spillover effects further, the multipliers for the country-specific variables are computed. Domestic and foreign variables can be expressed in terms of aggregate and relative variables.[20] Further, aggregate and relative fiscal changes are expressed as changes in home and foreign fiscal changes. Thus, cross-country transmission of fiscal policy can be analyzed.

[19]The relative fiscal multiplier depends on the three mentioned parameters positively. For the partial derivatives, see Appendix D.

[20]Note, that for a generic variable y the following identities hold: $y^H = y^W - (1-n)y^R$ and $y^F = y^W + ny^R$.

$$
\begin{aligned}
dx_t^H &= dx_t^W - (1-n)dx_t^R \\
&= \frac{1}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)}dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)-(1-n)(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^R \\
&= \frac{1+\gamma+\delta\lambda+(\gamma_x+\gamma_\pi\lambda)(1-n)\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^H + \frac{\gamma+\delta\lambda-(\gamma_x+\gamma_\pi\lambda)(1-n)\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^F \\
dx_t^F &= dx_t^W + ndx_t^R \\
&= \frac{1}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)}dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)+n(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^R \\
&= \frac{\gamma+\delta\lambda-(\gamma_x+\gamma_\pi\lambda)n\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^H + \frac{1+\gamma+\delta\lambda+(\gamma_x+\gamma_\pi\lambda)n\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}dg_t^F
\end{aligned}
$$

In the case of asymmetric country size, the implications of a fiscal expansion are not clear cut. To illustrate the complicated mechanism assume that the home government increases g_t^H, but the foreign country does not react. The fiscal expansion at home increases the demand for the home products directly and one-by-one. The foreign country benefits from the home boom via the trade channel, but by less than one-by-one (again, it is assumed that both γ and δ are positive). This leads to a decrease of the relative output gap.[21] So, competitiveness dampens the demand for the home products and decreases the output gap. Alternatively, because of the positive stimulus, inflation rates increase in both countries. As the foreign inflation rate increases by less than the home inflation rate, in total the relative inflation rate decreases. This dampens the home demand further via the nominal competitiveness channel. Moreover, the central bank responds to the immediate inflationary booms in both countries by an increase of the nominal interest rate via the Taylor rule.

In sum, the fiscal multiplier of home government spending is positive when γ and δ are positive.[22] The cross-country multiplier is positive if the parameters satisfy a certain parameter inequality.[23] If the home country is a small country, that is, n is small and the central bank assigns high coefficients to the Taylor rule, it is unlikely that the cross-country multiplier is positive. So, a foreign fiscal expansion leads to a reduction in the home output gap.

To get an idea of the magnitude of the multiplier, the model is calibrated. For the values of

[21] The structural equations (**??**) and (**??**) imply that the immediate response of the foreign output gap is an increase by $\frac{1}{1+\gamma}$ which implies that the relative output gap decreases at impact by $\frac{\gamma}{1+\gamma}$.

[22] In this case, the multiplier is positive when its nominator is positive which is true given the assumptions.

[23] This is given by $\gamma+\delta\lambda > (1-n)\varphi(\gamma_x+\gamma_\pi\lambda)$ given positive γ and δ.

the parameters see Table 1 of Appendix E. Furthermore, the coefficients of the Taylor rule are set to equal $\gamma_x = 0.5$, and $\gamma_\pi = 1.5$ to meet the Taylor principle. However, given that the model is highly stylized, these numbers should not be taken as policy advice. Table 3 reports the results for various combinations of the competitiveness parameters.

	$\gamma = 1/2$ $\delta = 1/2$				$\gamma = -1/2$ $\delta = 1/2$			
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	0.60	-0.03	0.48	0.12	0.60	-1.45	1.90	-1.30
dx_t^F	0.60	0.42	0.04	0.56	0.60	2.55	-2.09	2.69
dx_t^W	0.60	0.08	0.37	0.23	0.60	-0.45	0.9	-0.30
dx_t^R	0	0.44	-0.44	0.44	0	4	-4	4

	$\gamma = -1/2$ $\delta = -1/2$				$\gamma = 0$ $\delta = 0$			
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	0.60	1.75	-1.30	1.90	0.60	-0.25	0.70	-0.10
dx_t^F	0.60	-2.25	2.69	-2.09	0.60	0.75	-0.30	0.90
dx_t^W	0.60	0.75	-0.3	0.9	0.60	0	0.45	0.15
dx_t^R	0	-4	4	-4	0	1	-1	1

Table 3: Numerical multipliers when the Taylor rule applies

Last, the question is analyzed whether the central bank can completely offset a fiscal expansion by choosing appropriate values of the parameters of the Taylor rule. The question has to be negated. The multiplier of aggregate government spending is positive, albeit it can be very small when γ_x and γ_π are very large.

Even in the case when governments choose asymmetric fiscal stimulus packages such that on aggregate there is no fiscal change at all, but relative fiscal spending changes, then the central bank is not able to offset a fiscal change.[24]

5.3 The Multiplier With a ZLB Constraint

When the ZLB constraint is holding (which is equivalent to the case that the central bank follows a constant nominal interest rate rule) the fiscal multiplier changes. Then, as $d\bar{\iota} = 0$

[24]When the change of aggregate spending is zero, this implies that $dg_t^F = \frac{-n}{1-n} dg_t^H$, and $dg_t^R = \frac{-1}{1-n} dg_t^H$. Inserting this into the equations of the multiplier and assuming that the coefficients of dg_t^R for the change of both the home and foreign output gap are zero, leads to a contradictive conclusion.

equations (13) imply for the aggregate and the relative output gaps the following results

$$dx_t^W = dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} dg_t^R$$

$$dx_t^R = \frac{1}{1+2\gamma+2\delta\lambda} dg_t^R$$

Under the ZLB constraint the aggregate fiscal multiplier is always 1. However, the change of the aggregate output gap is determined by the the change of relative government spending as well. If both countries are of the same size, then the overall multiplier is given by 1. If both countries are asymmetric in country size, then intraunion spillovers play an important role in determining the size of the multiplier as in the previous scenario when the central bank follows a Taylor rule.

The relative government multiplier depends positively on the competitiveness parameters γ, and δ.[25] The mechanism is almost the same as in the previous section except the fact that the nominal interest rate does not increase under the ZLB. So, the expansionary effect of an increase in relative government spending is not dampened. As a result, the multiplier under the ZLB is bigger than the multiplier under the Taylor rule. To see this, the difference of the multiplier is computed

$$(dx_t^W)_{ZLB} - (dx_t^W)_{TR} = \frac{\gamma_x\varphi+\gamma_\pi\lambda\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} dg_t^W + \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} \frac{\gamma_x\varphi+\gamma_\pi\lambda\varphi}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} dg_t^R$$

The coefficient of the difference of both aggregate government spending multiplier is always positive implying that the multiplier under the ZLB is bigger than the one under the Taylor rule. The coefficient of the difference of both relative spending multipliers depends on the parameters n, γ and δ. When the home country is bigger than the neighbor country, the spillover coefficients satisfy the following constraint: either the sum $\gamma+\delta\lambda < -1/2$ or the sum $\gamma+\delta\lambda > 0$.[26]

To explore the spillover effects under the ZLB further, the changes of the domestic and foreign output gaps are computed. For the domestic and foreign variables the following results hold

$$dx_t^H = dx_t^W - (1-n)dx_t^R = dg_t^W + \left[n - \frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}\right] dg_t^R$$

$$dx_t^F = dx_t^W + n dx_t^R = dg_t^W + \left[n - \frac{\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}\right] dg_t^R$$

[25] For the partial derivatives see Appendix D which are all positive.

[26] This can be easily seen from the condition that the coefficient is positive if either both nominator and denominator are positive or both are negative.

Expressing these changes in terms of changes of home and foreign governmental changes implies

$$dx_t^H = \left[\frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}dg_t^H + \frac{\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}dg_t^F\right] \tag{15}$$

$$dx_t^F = \left[\frac{\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}dg_t^H + \frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}dg_t^F\right] \tag{16}$$

First, note that the effects of home and foreign government spending are symmetric to the output gaps and do not depend on the country size n, but on the spillover effects.
Regarding the change of the home output gap, the effect of home government spending is positive in two cases: either the sum $\gamma+\delta\lambda > -1/2$ or the sum $\gamma+\delta\lambda < -1$.[27] If the sum does not satisfy these conditions, that is, $-1 < \gamma+\delta\lambda < -1/2$, then the multiplier is even negative. To illustrate how the country-multiplier $\frac{dx^i}{dg^i}$, $i = H, F$ depends on the sum $\gamma+\delta\lambda$ see Figure 7. The multiplier jumps from a negative to a positive values when $\gamma+\delta\lambda = -1/2$.[28] At this point the model turns indeterminate.[29]

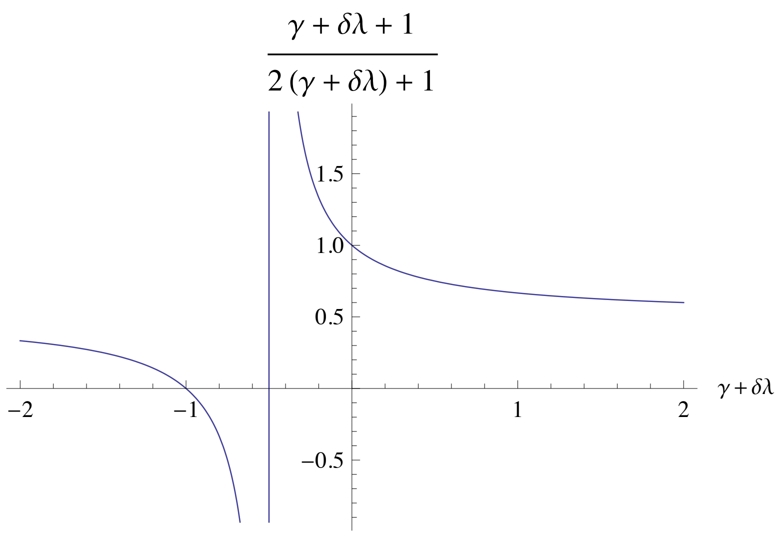

Figure 7: Graphical representation of the multiplier depending on the spillover effects I

On impact, an increase in home government spending leads to a boom in the home country by one-to-one. The real trade divergence measured by the relative output gap increases, but

[27]In the first case, both nominator and denominator of the multiplier are positive, whereas in the second case both are negative.

[28]Multipliers do not always depend linearly on underlying model parameters. For another example of a jump in a multiplier see Hall (2009).

[29]Eigenvalues of the system have the expression $1 + 2(\gamma+\delta\lambda)$ in their denominators which are 0 at the critical point.

suppose γ is negative, then this dampens home demand. Moreover, country-specific inflation rates increase, and nominal divergences increase as well, but assuming that δ is negative, the effect is a further contraction of home demand. In sum, all effects given the parameter constraint lead to a decrease of the home output gap.

Concerning the cross-country multipliers $\frac{dx^i}{dg^j}$, $i \neq j$ Figure 8 depicts the relation between the multiplier and the sum $\gamma + \delta\lambda$. This multiplier is negative if $-1/2 < \gamma + \delta\lambda < 0$. Otherwise it is positive.

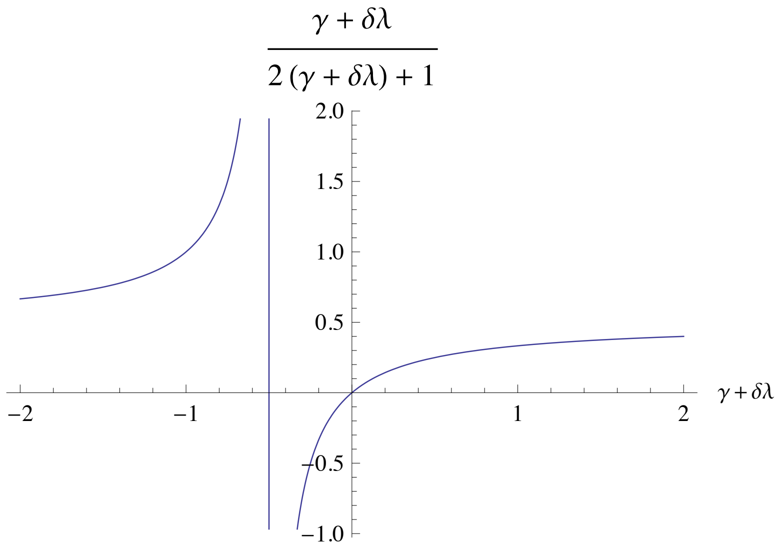

Figure 8: Graphical representation of the multiplier depending on the spillover effects II

Special cases arise when the sum $\gamma+\delta\lambda = -1$. Then the home government spending multiplier is zero, meaning that home fiscal policy does not serve as a stabilization tool in the home country. However, the cross-country spending multiplier is 1. So, a fiscal expansion abroad can help to stabilize the home economy. Second, when the sum $\gamma + \delta\lambda = -1/2$, then the coefficient of the home spending multiplier is infinite, and the cross-country multiplier is minus infinity.[30]

Table 4 reports values of the multipliers for several combinations of the competitiveness parameters to illustrate the mentioned facts.

According to Michalak et al. (2009) and van Aarle et al. (2004), econometric estimates of the spillover parameters imply that γ and δ are positive. So, in practice both multipliers are positive but less than one.

[30] To be more precise, this statement means that when the sum is in a small neighborhood of the value -1/2, but greater than this value, the multiplier gets infinite resp. minus infinite. Note that $\lim_{x\to-1/2} = \frac{1+x}{1+2x} = \infty$ and $\lim_{x\to-1/2} = \frac{x}{1+2x} = -\infty$.

	$\gamma = 1/2$	$\delta = 1/2$			$\gamma = -1/2$	$\delta = 1/2$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	1	0.03	0.72	0.28	1	-1.75	2.5	-1.5
dx_t^F	1	0.47	0.28	0.72	1	2.25	-1.5	2.5
dx_t^W	1	0.14	0.61	0.39	1	-0.75	1.5	-0.5
dx_t^R	0	0.44	-0.44	0.44	0	4	-4	4
	$\gamma = -1/2$	$\delta = -1/2$			$\gamma = 0$	$\delta = 0$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	1	2.25	-1.5	2.5	1	-0.25	1	0
dx_t^F	1	-1.75	2.5	-1.5	1	0.75	0	1
dx_t^W	1	1.25	-0.5	1.5	1	0	0.75	0.25
dx_t^R	0	-4	4	-4	0	1	-1	1

Table 4: Numerical multipliers when the ZLB binds

Comparing the results under the ZLB constraint and the results obtained under the Taylor rule yields

$$(dx_t^H)_{ZLB} - (dx_t^H)_{TR} = \frac{(\gamma_x + \gamma_\pi \lambda)\varphi}{1 + \gamma_x \varphi + \gamma_\pi \lambda \varphi} dg_t^W + \frac{(\gamma_x + \gamma_\pi \lambda)(\gamma + \delta\lambda)(2n-1)\varphi}{(1 + 2\gamma + 2\delta\lambda)(1 + \gamma_x \varphi + \gamma_\pi \lambda\varphi)} dg_t^R$$

$$(dx_t^F)_{ZLB} - (dx_t^F)_{TR} = \frac{(\gamma_x + \gamma_\pi(\lambda))\varphi}{1 + \gamma_x \varphi + \gamma_\pi \lambda \varphi} dg_t^W + \frac{(\gamma_x + \gamma_\pi \lambda)(\gamma + \delta\lambda)(2n-1)\varphi}{(1 + 2\gamma + 2\delta\lambda)(1 + \gamma_x \varphi + \gamma_\pi \lambda\varphi)} dg_t^R$$

The coefficient of the difference of the aggregate government spending multiplier is always positive. This implies that the multiplier of aggregate government spending under the ZLB is always bigger than the one under the Taylor rule. The coefficient of the difference of relative government spending depends, as indicated earlier on the country size n and γ and δ. Different combination of these parameters lead to a positive or a negative coefficient.[31]

5.4 Multipliers and Implementation Lags in Fiscal Policy

In the aforementioned analysis, it is assumed that governments decide on the stimulus packages and that these can be implemented without any delay. So, increases in government spending have an immediate effect on the variables. This assumption is implausible in practice. Governments have to debate about stimulus packages in parliament: politicians might

[31] The following cases have to be distinguished: First, $n > 1/2$. Then the coefficient is positive if both the nominator and denominator are positive which is equivalent to the fact that the sum $\gamma + \delta\lambda > 0$, or both are negative which is the case when the sum is less than $-1/2$. Second, $n < 1/2$. The coefficient is positive if $-1/2 < \gamma + \delta\lambda < 0$. Note, that the terms involving the Taylor rule coefficients are positive.

have to decide about legislative changes. Discretionary fiscal policy is not effective as soon as it is implemented as automatic stabilizers. In sum, from the time when governments recognize that there is a shock hitting the economy which has to be stabilized until the time fiscal policy starts to have an effect on the output gap, recognition, decision and implementation lags play an important role.

In contrast, monetary policy can react faster than can fiscal policy in normal times. However, with the short-term nominal interest rate being at zero the central bank cannot change the short-term policy rate to accommodate fiscal policy for some time.

To account for the delay of the implementation of fiscal policy in this section, I assume that fiscal policy decides upon a stimulus plan of government spending at the beginning of period t, which lasts for three periods from time t to $t+2$. Moreover, as in **?**, assuming that government spending follows an $AR(2)$-process to account for implementation lags in fiscal policy, that is, under the assumption that $G_t = \rho_1 G_{t-1} + \rho_2 G_{t-2} + \epsilon_t$ where ρ_1 and ρ_2 are constants and ϵ_t is an i.i.d. shock with mean zero, then the one-period multiplier is given by

$$\frac{dX_t}{dG_t} = (B + AB\rho_1 + A^2 B(\rho_1^2 + \rho_2)). \tag{17}$$

The central bank is still bound by zero nominal interest rate. The model with the matrices A and B is derived in Appendix F.

Table 5 reports numerical values for the various multipliers when there are implementation lags in fiscal policy, assuming that $\rho_1 = 0.75$, but $\rho_2 = 0$.[32]

To summarize, under the ZLB and considering implementation lags in fiscal policy, governmental multipliers are large size compared with the other cases discussed earlier. The multipliers even increase more when implementation lags increase. Table 8 illustrates this issue under the additional assumption that $\rho_2 = 0.75$.

6 Conclusion

Using a two-country New Keynesian model of a monetary union the interaction of fiscal and monetary policies are analyzed. While the central bank is constrained by ZLB on the nominal interest rate, both governments are free to adjust their instruments to stabilize different kind of shocks. The outcome of coordinated fiscal policies to maximize a weighted average of country-specific plus a monetary welfare functions as a benchmark scenario is

[32]The values for the matrix algebra AB or A^2B are not reported here. The results are derived using the program Mathematica 7, and the program files are available upon request.

	$\gamma = 1/2$	$\delta = 1/2$			$\gamma = -1/2$	$\delta = 1/2$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	2.79	0.39	1.70	1.08	2.79	-22.58	24.67	-21.88
dx_t^F	2.79	1.01	1.08	1.70	2.79	23.97	-21.88	24.67
dx_t^W	2.79	0.54	1.55	1.23	2.79	-10.94	13.03	-10.24
dx_t^R	0	0.62	-0.62	0.62	0	46.55	-46.55	46.55
	$\gamma = -1/2$	$\delta = -1/2$			$\gamma = 0$	$\delta = 0$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	2.79	29.21	-27.12	29.91	2.79	-0.7	2.79	0
dx_t^F	2.79	-27.82	29.91	-27.12	2.79	2.09	0	2.79
dx_t^W	2.79	14.95	-12.85	15.65	2.79	0	2.09	0.7
dx_t^R	0	-57.03	57.03	-57.03	0	2.79	-2.79	2.79

Table 5: Numerical multipliers when fiscal policy suffers from implementation lags, $\rho_1 = 0.75$, $\rho_2 = 0$.

computed and compared with the case of a Nash game of simultaneous setting of country-specific governmental spending. In both scenarios monetary policy does not react as it is assumed that the ZLB constraint binds.

It is assumed that the monetary union is in a situation that the nominal interest rate is zero, and that it is hit by several different shocks (cost-push or demand shocks). It depends on the kind of shocks whether countries prefer to coordinate or not to coordinate their fiscal policy.

In a second step, government spending multipliers when monetary policy is optimal or follows a Taylor rule are computed and compared with the values of the multipliers when the nominal interest rate is zero. The size of the various multipliers depends on the combination of the intraunion competitiveness parameters and is not necessarily large when the ZLB constraint binds. However, the fiscal effects are amplified when there are implementation lags in government spending.

One of the key missing elements is an endogenous time length of the ZLB constraint. In this paper, the focus is to analyze the problem how governments should interact while nominal interest rates are binding. To find an answer to the question of how fiscal policy can react to terminate the ZLB constraint and stabilize the economy is left for future research.

Second, as the first fiscal stimulus packages have phased out and the economy has started to recover, the question turns to fiscal consolidation packages. How can deficits be avoided such that the economy does not turn into a recession again? Thus, debt dynamics should be included into the analysis.

A The model in matrices

The system of structural equations can be rearranged to yield the following representation:

$$
\begin{aligned}
\begin{pmatrix} x_t^H \\ x_t^F \\ \pi_t^H \\ \pi_t^F \end{pmatrix} &= C^{-1} \begin{pmatrix} 1 & 0 & \varphi & 0 \\ 0 & 1 & 0 & \varphi \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} E_t \begin{pmatrix} x_{t+1}^H \\ x_{t+1}^F \\ \pi_{t+1}^H \\ \pi_{t+1}^F \end{pmatrix} \\
&+ \frac{1}{1+2\gamma+2\delta\lambda} \begin{pmatrix} -\varphi(1+2\gamma+2\delta\lambda) & (1+\gamma+\delta\lambda) & \gamma+\delta(\lambda) \\ -\varphi(1+2\gamma+2\delta\lambda) & \gamma+\delta(\lambda) & (1+\gamma+\delta\lambda) \\ -\varphi\lambda(1+2\gamma+2\delta\lambda) & \lambda(1+\gamma+\delta\lambda) & \lambda(\gamma+\delta(\lambda)) \\ -\varphi\lambda(1+2\gamma+2\delta\lambda) & \lambda(\gamma+\delta(\lambda)) & \lambda(1+\gamma+\delta\lambda) \end{pmatrix} \begin{pmatrix} \bar{\iota}_t \\ g_t{}^H \\ g_t{}^F \end{pmatrix} + A^{-1} \begin{pmatrix} \varepsilon_t^H \\ \varepsilon_t^F \\ u_t^H \\ u_t^F \end{pmatrix} \qquad (18)
\end{aligned}
$$

where the matrix C^{-1} is given by

$$
C^{-1} = \frac{1}{1+2\gamma+2\delta\lambda} \begin{pmatrix} 1+\gamma+\delta\lambda & \gamma+\delta\lambda & -\delta & \delta \\ \gamma+\delta\lambda & 1+\gamma+\delta\lambda & \delta & -\delta \\ \lambda(1+\gamma+\delta\lambda) & (\gamma+\delta\lambda)\lambda & 1+2\gamma+\delta\lambda & \delta\lambda \\ (\gamma+\delta\lambda)\lambda & \lambda(1+\gamma+\delta\lambda) & \delta\lambda & 1+2\gamma+\delta\lambda \end{pmatrix}
$$

Aggregate and relative equations of the model are given by

$$
\begin{aligned}
\pi_t^W &= \beta E_t \pi_{t+1}^W + \lambda x_t^W + u_t^W \\
x_t^W &= E_t x_{t+1}^W - \varphi(\bar{\iota}_t - E_t \pi_{t+1}^W) + g_t^W + \gamma(2n-1)x_t^R + \delta(2n-1)\pi_t^R + \varepsilon_t^W \\
\pi_t^R &= \beta E_t \pi_{t+1}^R + \lambda x_t^R + u_t^R \\
(1+2\gamma)x_t^R &= E_t x_{t+1}^R + \varphi E_t \pi_{t+1}^R + g_t^R - 2\delta\pi_t^R + \varepsilon_t^R
\end{aligned}
$$

These equations can be cast into matrix form

$$
\begin{pmatrix} 1 & -\lambda & 0 & 0 \\ 0 & 1 & -\delta(2n-1) & -\gamma(2n-1) \\ 0 & 0 & 1 & -\lambda \\ 0 & 0 & 2\delta & 1+2\gamma \end{pmatrix} \begin{pmatrix} \pi_t^W \\ x_t^W \\ \pi_t^R \\ x_t^R \end{pmatrix}
$$

$$
= \begin{pmatrix} \beta & 0 & 0 & 0 \\ \varphi & 1 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \varphi & 1 \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1}^W \\ x_{t+1}^W \\ \pi_{t+1}^R \\ x_{t+1}^R \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ -\varphi & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{\iota}_t \\ g_t{}^W \\ g_t{}^R \end{pmatrix} + \begin{pmatrix} u_t^W \\ \varepsilon_t^W \\ u_t^R \\ \varepsilon_t^R \end{pmatrix}
$$

Applying the inverse B^{-1} of the coefficient matrix on both sides of the system yields

$$\begin{pmatrix} \pi_t^W \\ x_t^W \\ \pi_t^R \\ x_t^R \end{pmatrix} = B^{-1} \begin{pmatrix} \beta & 0 & 0 & 0 \\ \varphi & 1 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \varphi & 1 \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1}^W \\ x_{t+1}^W \\ \pi_{t+1}^R \\ x_{t+1}^R \end{pmatrix}$$

$$+ \frac{1}{1+2\gamma+2\delta\lambda} \begin{pmatrix} -\lambda\varphi(1+2\gamma+2\delta\lambda) & (\lambda)(1+2\gamma+2\delta\lambda) & (2n-1)\lambda(\gamma+\delta(\lambda)) \\ -\varphi(1+2\gamma+2\delta\lambda) & (1+2\gamma+2\delta\lambda) & (2n-1)(\gamma+\delta(\lambda)) \\ 0 & 0 & \lambda \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{\iota}_t \\ {g_t}^W \\ {g_t}^R \end{pmatrix} \quad (19)$$

$$+ B^{-1} \begin{pmatrix} u_t^W \\ \varepsilon_t^W \\ u_t^R \\ \varepsilon_t^R \end{pmatrix}$$

B The system in normal times when the central bank follows a Taylor rule

Assuming that in normal times monetary policy follows a Taylor rule, the central bank sets the nominal interest rate according to

$$\bar{\iota} = \gamma_\pi \pi_t^W + \gamma_x x_t^W$$

where the coefficients $\gamma_x > 0$ and $\gamma_\pi > 1$ to follow the Taylor principle.

Inserting this interest rate rule into the aggregate demand equation and solving for the endogenous variables leads to the following algebra

$$\begin{pmatrix} 1 & -\lambda & 0 & 0 \\ \varphi\gamma_\pi & 1+\varphi\gamma_x & -\delta(2n-1) & -\gamma(2n-1) \\ 0 & 0 & 1 & -\lambda \\ 0 & 0 & 2\delta & 1+2\gamma \end{pmatrix} \begin{pmatrix} \pi_t^W \\ x_t^W \\ \pi_t^R \\ x_t^R \end{pmatrix}$$

$$= \begin{pmatrix} \beta & 0 & 0 & 0 \\ \varphi & 1 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \varphi & 1 \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1}^W \\ x_{t+1}^W \\ \pi_{t+1}^R \\ x_{t+1}^R \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} {g_t}^W \\ {g_t}^R \end{pmatrix} + \begin{pmatrix} u_t^W \\ \varepsilon_t^W \\ u_t^R \\ \varepsilon_t^R \end{pmatrix}$$

The inverse of the matrix of coefficients on the left hand side is given by

$$C^{-1}=\begin{pmatrix} \frac{1+\gamma_x\varphi}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi} & \frac{\lambda}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi} & \frac{(2n-1)\delta\lambda}{(1+2\gamma+2\delta\lambda)(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} & \frac{\lambda(\gamma+\delta\lambda)(2n-1)}{(1+2\gamma+2\delta\lambda)(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} \\ \frac{-\gamma_\pi\varphi}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi} & \frac{1}{1+\gamma_x\varphi+\gamma_\pi\lambda\varphi} & \frac{\delta(2n-1)}{(1+2\gamma+2\delta\lambda)(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} & \frac{(2n-1)(\gamma+\delta\lambda)}{(1+2\gamma+2\delta\lambda)(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)} \\ 0 & 0 & \frac{1+2\gamma}{1+2\gamma+2\delta\lambda} & \frac{\lambda}{1+2\gamma+2\delta\lambda} \\ 0 & 0 & \frac{-2\delta}{1+2\gamma+2\delta\lambda} & \frac{1}{1+2\gamma+2\delta\lambda} \end{pmatrix}$$

Applying the inverse C^{-1} of the matrix of coefficients C on the left hand side on both sides of the system yields

$$\begin{pmatrix} \pi_t^W \\ x_t^W \\ \pi_t^R \\ x_t^R \end{pmatrix} = C^{-1}\begin{pmatrix} \beta & 0 & 0 & 0 \\ \varphi & 1 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & \varphi & 1 \end{pmatrix} E_t \begin{pmatrix} \pi_{t+1}^W \\ x_{t+1}^W \\ \pi_{t+1}^R \\ x_{t+1}^R \end{pmatrix}$$

$$+\frac{1}{(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi)(1+2\gamma+2\delta\lambda)}\begin{pmatrix} \lambda(1+2\gamma+2\delta\lambda) & (2n-1)\lambda(\gamma+\delta(\lambda)) \\ (1+2\gamma+2\delta\lambda) & (2n-1)(\gamma+\delta(\lambda)) \\ 0 & \lambda(1+\gamma_x\varphi+\gamma_\pi\lambda\varphi) \\ 0 & (1+\gamma_x\varphi+\gamma_\pi\lambda\varphi) \end{pmatrix}\begin{pmatrix} {g_t}^W \\ {g_t}^R \end{pmatrix} \tag{20}$$

$$+C^{-1}\begin{pmatrix} u_t^W \\ \varepsilon_t^W \\ u_t^R \\ \varepsilon_t^R \end{pmatrix}$$

C Welfare effects of the policy scenarios

To rank the different policy regimes of full coordination and noncoordination the losses due to a home demand and a home cost-push shock are computed. Values for the aggregate loss function Agg (9), the monetary loss function CB (5), and the country-specific H and F fiscal loss functions (3) are computed for the standard calibration as given in table 1 but with different values for δ and n. For the losses $\delta=-1/2$ and n=1/2.

	Home cost-push				Home demand shock			
	Agg	CB	H	F	Agg	CB	H	F
Benchmark	700	649	15.8	86.44	6.68	0.77	5.91	5.91
Nash	1300	1200	16.215	116.40	2.846	0.129	5.43	0.0038

Table 6: Welfare losses (one-period) as a result of various shocks. Numbers have to be multiplied by 10^{-6}.

D Derivatives of the multipliers

In this section the partial derivatives of the multipliers are computed to see the dependence of the multipliers on the several parameters.

One of the multipliers is given by

$$a \equiv \frac{1+\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}$$

Partial derivatives of this multiplier with respect to the several parameters are given by

$$\frac{\partial a}{\partial \gamma} = \frac{-1}{(1+2\gamma+2\delta\lambda)^2}$$

$$\frac{\partial a}{\partial \delta} = \frac{-\lambda}{(1+2\gamma+2\delta\lambda)^2}$$

$$\frac{\partial a}{\partial \lambda} = \frac{-\delta}{(1+2\gamma+2\delta\lambda)^2}$$

The partial derivatives of the multiplier

$$b \equiv \frac{\gamma+\delta\lambda}{1+2\gamma+2\delta\lambda}$$

with respect to the several parameters are given by

$$\frac{\partial b}{\partial \gamma} = \frac{1}{(1+2\gamma+2\delta\lambda)^2}$$

$$\frac{\partial b}{\partial \delta} = \frac{\lambda}{(1+2\gamma+2\delta\lambda)^2}$$

$$\frac{\partial b}{\partial \lambda} = \frac{\delta}{(1+2\gamma+2\delta\lambda)^2}$$

E Numerical multipliers

Assume that in general the multipliers are given by the values a_{ij}, $i, j = 1, \ldots 4$, as indicated in the following table 7.

	$\gamma = \ldots$		$\delta = \ldots$	
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H	a_{11}	a_{12}	a_{13}	a_{14}
dx_t^F	a_{21}	a_{22}	a_{23}	a_{24}
dx_t^W	a_{31}	a_{32}	a_{33}	a_{34}
dx_t^R	a_{41}	a_{42}	a_{43}	a_{44}

Table 7: General multipliers.

Then the following relationships between the multipliers hold for each row $i = 1, \ldots 4$:

$$\begin{aligned} a_{i1} &= a_{i3} + a_{i4} \\ a_{i2} &= n a_{i1} - a_{i3} \\ a_{i3} &= -a_{i2} + n a_{i1} \\ a_{i4} &= a_{i2} + (1-n) a_{i1} \end{aligned}$$

For each column $j = 1, \ldots 4$ the equations hold

$$\begin{aligned} a_{3j} &= n a_{1j} + (1-n) a_{2j} \\ a_{4j} &= a_{2j} - a_{1j} \end{aligned}$$

F Numerical multipliers and implementation lags in fiscal policy

Still assuming that the nominal interest rate is equal to zero, the system of the aggregate and relative variables can be rearranged to

$$\begin{pmatrix} \pi_t^W \\ x_t^W \\ \pi_t^R \\ x_t^R \end{pmatrix} = A\, E_t \begin{pmatrix} \pi_{t+1}^W \\ x_{t+1}^W \\ \pi_{t+1}^R \\ x_{t+1}^R \end{pmatrix} + B \begin{pmatrix} g_t{}^W \\ g_t{}^R \end{pmatrix} + C \begin{pmatrix} u_t^W \\ \varepsilon_t^W \\ u_t^R \\ \varepsilon_t^R \end{pmatrix} \tag{21}$$

where the matrices are given by

$$A = \begin{pmatrix} \beta+\lambda\varphi & \lambda & \frac{(2n-1)\lambda(\beta\delta+(\gamma+\delta\lambda)\varphi)}{1+2\gamma+2\delta\lambda} & \frac{(2n-1)\lambda(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} \\ \varphi & 1 & \frac{(2n-1)(\beta\delta+(\gamma+\delta\lambda)\varphi)}{1+2\gamma+2\delta\lambda} & \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} \\ 0 & 0 & \frac{\beta(1+2\gamma)+\lambda\varphi}{1+2\gamma+2\delta\lambda} & \frac{\lambda}{1+2\gamma+2\delta\lambda} \\ 0 & 0 & \frac{-2\beta\delta+\varphi}{1+2\gamma+2\delta\lambda} & \frac{1}{1+2\gamma+2\delta\lambda} \end{pmatrix}$$

$$B = \begin{pmatrix} \lambda & \frac{(2n-1)\lambda(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} \\ 1 & \frac{(2n-1)(\gamma+\delta\lambda)}{1+2\gamma+2\delta\lambda} \\ 0 & \frac{\lambda}{1+2\gamma+2\delta\lambda} \\ 0 & \frac{1}{1+2\gamma+2\delta\lambda} \end{pmatrix}$$

The system of the counry-specific variables can be rearranged to

$$\begin{pmatrix} x_t^H \\ x_t^F \\ \pi_t^H \\ \pi_t^F \end{pmatrix} = A_1\, E_t \begin{pmatrix} x_{t+1}^H \\ x_{t+1}^F \\ \pi_{t+1}^H \\ \pi_{t+1}^F \end{pmatrix} + B_1 \begin{pmatrix} {g_t}^H \\ {g_t}^F \end{pmatrix} + C_1 \begin{pmatrix} \varepsilon_t^H \\ \varepsilon_t^F \\ u_t^H \\ u_t^F \end{pmatrix}$$

where the matrices are given by

$$A_1 = \frac{1}{1+2\gamma+2\delta\lambda} \begin{pmatrix} 1+\gamma+\delta\lambda & \gamma+\delta\lambda & -\beta\delta+(1+\gamma+\delta\lambda)\varphi & \beta\delta+(\gamma+\delta\lambda)\varphi \\ \gamma+\delta\lambda & 1+\gamma+\delta\lambda & \beta\delta+(\gamma+\delta\lambda)\varphi & -\beta\delta+(1+\gamma+\delta\lambda)\varphi \\ \lambda(1+\gamma+\delta\lambda) & \lambda(\gamma+\delta\lambda) & \beta(1+2\gamma+\delta\lambda)+\lambda(1+\gamma+\delta\lambda)\varphi & \lambda(\beta\delta+(\gamma+\delta\lambda)\varphi) \\ \lambda(\gamma+\delta\lambda) & \lambda(1+\gamma+\delta\lambda) & \lambda(\beta\delta+(\gamma+\delta\lambda)\varphi) & \beta(1+2\gamma+\delta\lambda)+\lambda(1+\gamma+\delta\lambda)\varphi \end{pmatrix}$$

$$B_1 = \frac{1}{1+2\gamma+2\delta\lambda} \begin{pmatrix} 1+\gamma+\delta\lambda & \gamma+\delta\lambda \\ \gamma+\delta\lambda & 1+\gamma+\delta\lambda \\ \lambda(1+\gamma+\delta\lambda) & \lambda(\gamma+\delta\lambda) \\ \lambda(\gamma+\delta\lambda) & \lambda(1+\gamma+\delta\lambda) \end{pmatrix}$$

Assuming that government spending follows an $AR(2)$-process to account for implementation lags in fiscal policy, i.e., under the assumption that $G_t = \rho_1 G_{t-1} + \rho_2 G_{t-2} + \epsilon_t$ where ρ_1 and ρ_2 are constants and ϵ_t is an i.i.d. shock with mean zero, then the one-period multiplier is given by

$$\frac{dX_t}{dG_t} = (B + AB\rho_1 + A^2 B(\rho_1^2 + \rho_2)).$$

Algebraic expressions for the matrices of this equation and thus for the multipliers are derived using the programme Mathematica 7. Due to space constraints this multipliers are not reported in general. Programme files are available upon request.

Table 5 in section 5.4 reports numerical values for the various multipliers when there are implementation lags in fiscal policy, assuming that $\rho_1 = 0.75$, but $\rho_2 = 0$.

The next table 8 gives values in the case that $\rho_1 = 0.75$, but $\rho_2 = 0.75$.

	$\gamma = 1/2$	$\delta = 1/2$			$\gamma = -1/2$	$\delta = 1/2$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H			2.32	1.66			46.12	-42.13
dx_t^F			1.66	2.32			-42.13	46.12
dx_t^W	3.98	0.83			3.98	-21.07		
dx_t^R	0	0.67			0	88.25		
	$\gamma = -1/2$	$\delta = -1/2$			$\gamma = 0$	$\delta = 0$		
Variable	dg_t^W	dg_t^R	dg_t^H	dg_t^F	dg_t^W	dg_t^R	dg_t^H	dg_t^F
dx_t^H			-73.36	77.34			3.98	0
dx_t^F			77.34	-73.36			0	3.98
dx_t^W	3.98	38.67			3.98	0		
dx_t^R	0	-150.71			0	3.98		

Table 8: Numerical multipliers when fiscal policy suffers from implementation lags, $\rho_1 = 0.75$, $\rho_2 = 0.75$.

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